Constraints

Engineering Mechanics: Dynamics

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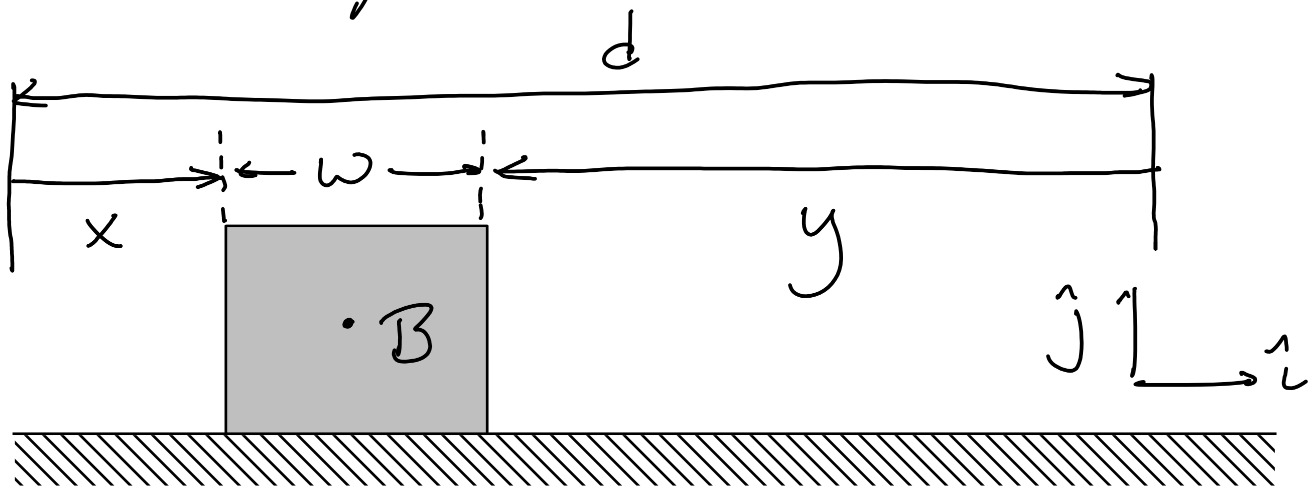
Kinematics: Consideration of the allowable motion of an object, consistent with the constraints that act on that object, without regard to the forces that produce motion or the motion that actually occurs

- RESTRICTIONS ON THE ALLOWABLE MOTION
- IMPOSE A RELATIONSHIP BETWEEN COORDINATES



$$x(t) + w + y(t) = d$$

$$y(t) = (d - w) - x(t)$$

CONSTRAINTS CAN BE IDENTIFIED FROM VELOCITIES

$$\underline{v}_B = \dot{x}\,\hat{i} = -\dot{y}\,\hat{i}$$

$$\int_0^t \left\{ \dot{x} = -\dot{y} \right\} dt$$

$$x(t) - x(0) = -\left(y(t) - y(0) \right)$$

$$y(t) = -x(t) + \left(y(0) + x(0) \right) = -x(t) + (d - w)$$

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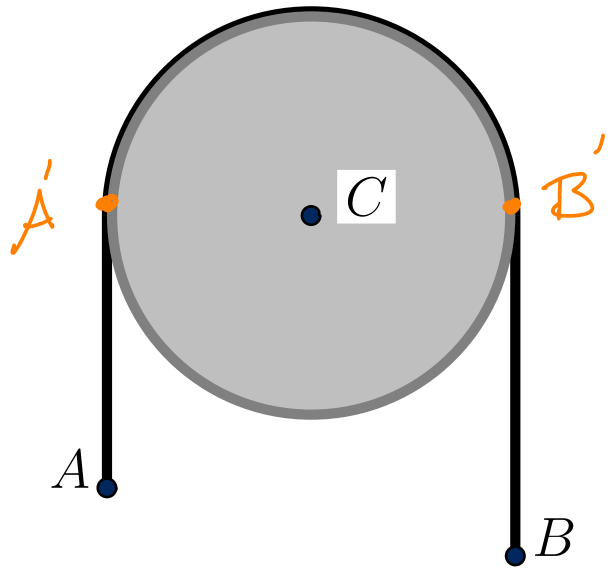
Pulleys

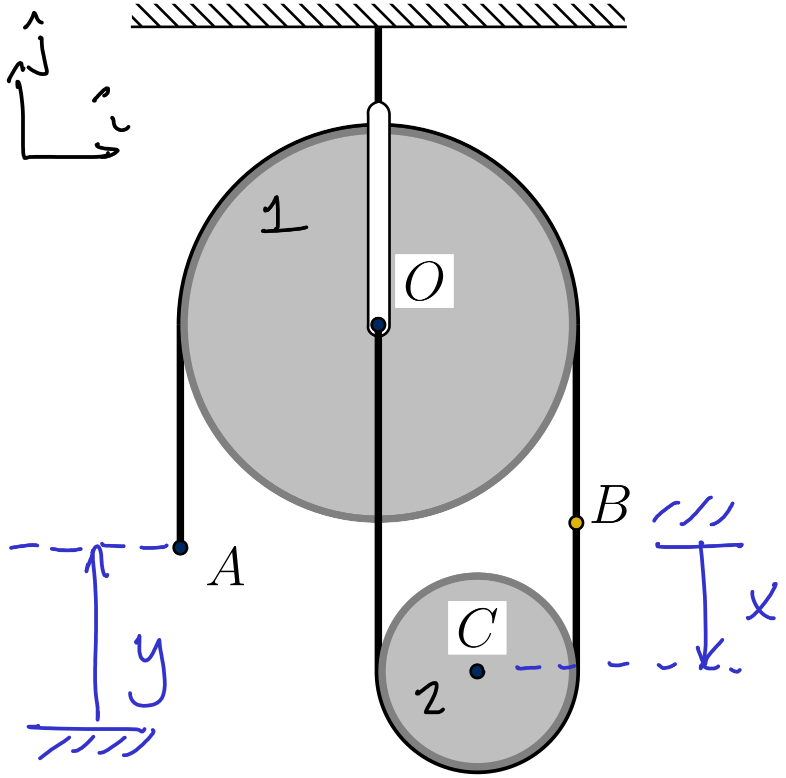
$$\underline{v}_A = \underline{v}_C + \underline{\omega}_P \times \underline{r}_{A/C}$$

$$\underline{v}_B = \underline{v}_C + \underline{\omega}_P \times \underline{r}_{B/C}$$

$$\underline{r}_{A/C} = -\underline{r}_{B/C}$$



$$\underline{v}_A + \underline{v}_B = 2\underline{v}_C \qquad \underline{v}_B - \underline{v}_A = 2\underline{\omega}_P \times \underline{r}_{B/C}$$



ON 1:

$$\underline{v}_A + \underline{v}_B = 2\cancel{\underline{v}_O}^{\,0} \qquad \cancel{\underline{v}_O}^{\,0} + \underline{v}_B = 2\underline{v}_C$$

$$\underline{v}_A + (2\underline{v}_C) = \underline{0} \longrightarrow \underline{v}_C = -\frac{\underline{v}_A}{2}$$

$$(-\dot{x}\hat{j}) = -\frac{(\dot{y}\hat{j})}{2}$$

$$\dot{x} = \frac{\dot{y}}{2} \longrightarrow \boxed{x = \frac{y}{2}}$$

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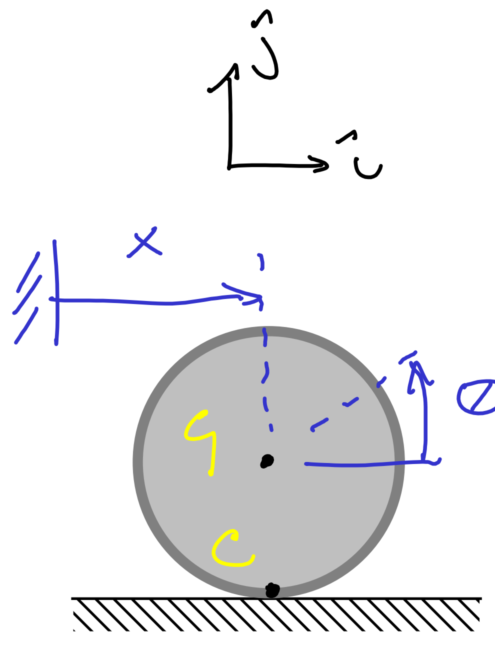
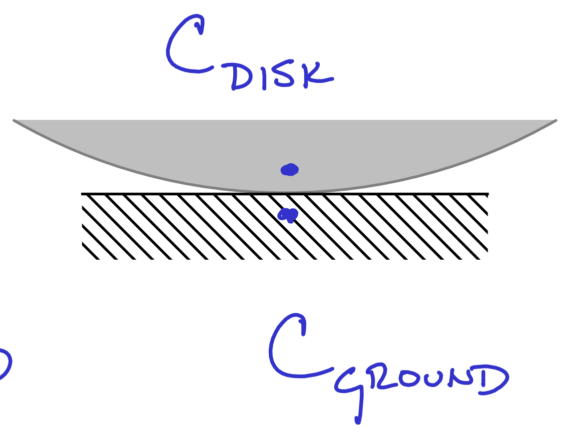
Rolling

NO SLIP IMPLIES THE CONTACT POINTS HAVE THE SAME VELOCITY

$$\underline{v}_{C_{DISK}} = \underline{v}_{C_{GROUND}} = \underline{0}$$





$$\underline{v}_g = \underline{v}_c + \underline{\omega}_D \times \underline{r}_{g/c}$$

$$(\dot{x}\,\hat{\imath}) = \underline{0} + (\dot{\varnothing}\,\hat{k}) \times (r\,\hat{\jmath})$$

$$= (-r\dot{\varnothing})\,\hat{\imath}$$

$$\dot{x} = -r\dot{\varnothing}$$

$$\int_0^t \left\{ \dot{x} = -r\dot{\varnothing} \right\} dt$$

$$x(t) - x(0) = -r\,(\varnothing(t) - \varnothing(0))$$

$$x(t) = -r\varnothing(t) + (x(0) + r\varnothing(0))$$

MEASURING FROM A COMMON CONFIGURATION

$$x(0) + r\varnothing(0) = 0$$

$$\boxed{x(t) = -r\varnothing(t)}$$

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