Constraints

Engineering Mechanics: Dynamics

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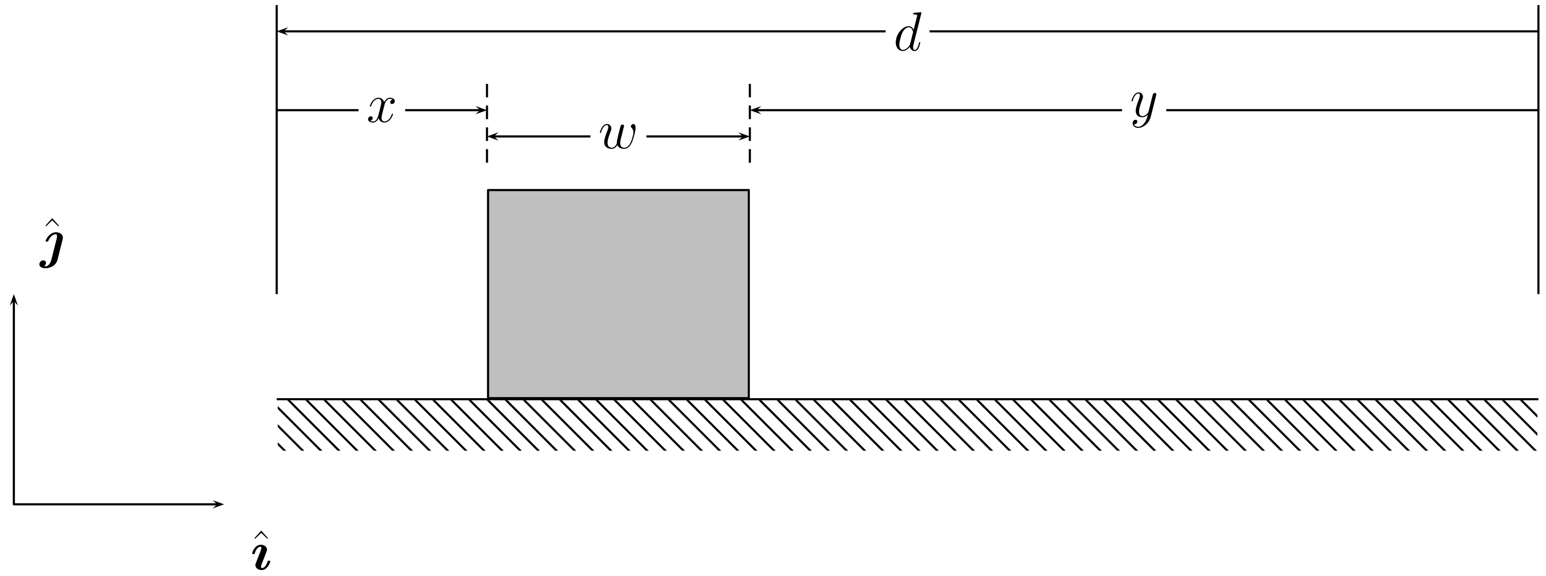
Kinematics: Consideration of the allowable motion of an object, consistent with the constraints that act on that object, without regard to the forces that produce motion or the motion that actually occurs

Constraints describe

- restriction on the allowable motion
- impose a relationship between two or more coordinates

The position of the block can be described by either $x(t)$ or $y(t)$, and

$$x(t) + w + y(t) = d, \quad \longrightarrow$$
$$y(t) = (d - w) - x(t).$$



The constraints can also be identified from the velocity of the block

$$\dot{x}\,\hat{\boldsymbol{\imath}} = \underline{\mathbf{v}}_B = -\dot{y}\,\hat{\boldsymbol{\imath}}, \quad \longrightarrow \quad \int_0^t \left\{ \dot{x} = -\dot{y} \right\} dt,$$
$$x(t) - x(0) = -\left(y(t) - y(0)\right),$$
$$\longrightarrow \quad y(t) = -x(t) + (y(0) + x(0)),$$
$$= -x(t) + (d - w)$$

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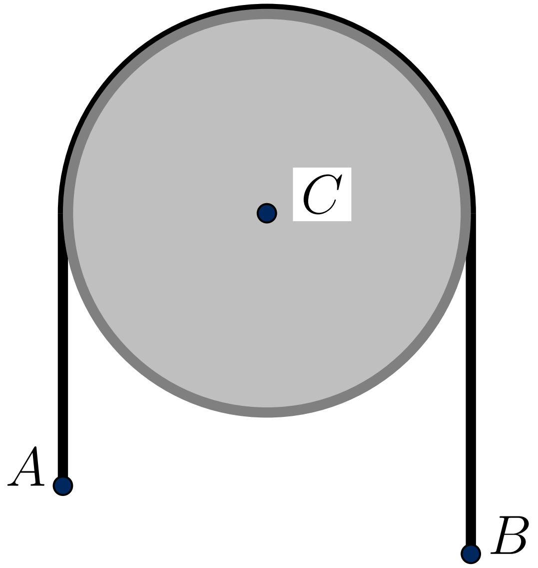
Pulleys

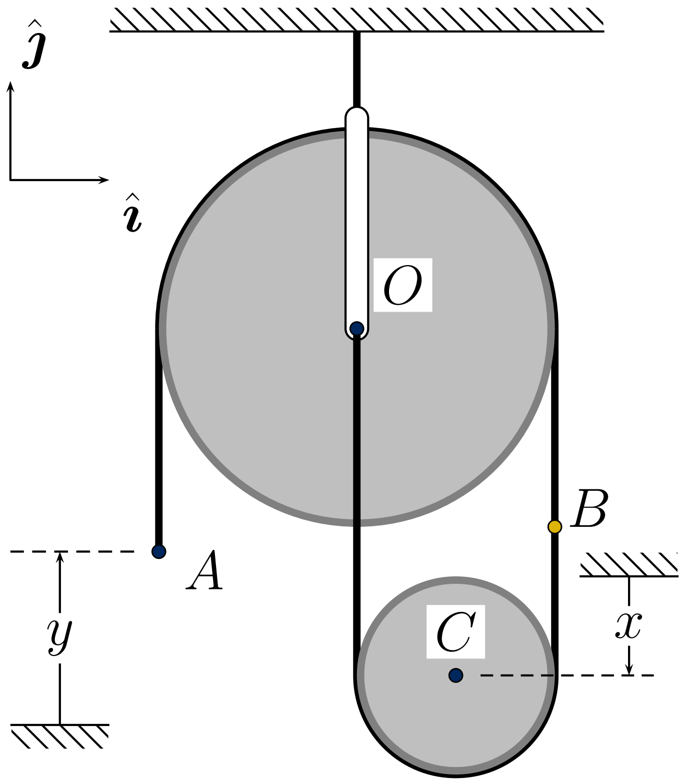
The velocities of the edges and center of a pulley can be related as



$$\underline{\mathbf{v}}_A = \underline{\mathbf{v}}_C + \underline{\boldsymbol{\omega}}_{\mathcal{P}} \times \underline{\mathbf{r}}_{A/C},$$
$$\underline{\mathbf{v}}_B = \underline{\mathbf{v}}_C + \underline{\boldsymbol{\omega}}_{\mathcal{P}} \times \underline{\mathbf{r}}_{B/C},$$

since $\underline{\mathbf{r}}_{A/C} = -\underline{\mathbf{r}}_{B/C}$

$$\underline{\mathbf{v}}_A + \underline{\mathbf{v}}_B = 2\,\underline{\mathbf{v}}_C, \qquad \underline{\mathbf{v}}_B - \underline{\mathbf{v}}_A = 2\,\underline{\boldsymbol{\omega}}_{\mathcal{P}} \times \underline{\mathbf{r}}_{B/C}$$



For a pulley system consider each pulley separately

$$\underline{\mathbf{v}}_A + \underline{\mathbf{v}}_B = 2\,\underline{\mathbf{v}}_O, \qquad \underline{\mathbf{v}}_O + \underline{\mathbf{v}}_B = 2\,\underline{\mathbf{v}}_C$$

since $\underline{\mathbf{v}}_O = \underline{\mathbf{0}}$

$$\underline{\mathbf{v}}_A + 2\,\underline{\mathbf{v}}_C = \underline{\mathbf{0}}, \quad \longrightarrow \quad \underline{\mathbf{v}}_C = -\frac{\underline{\mathbf{v}}_A}{2},$$
$$(-\dot{x}\,\hat{\boldsymbol{\jmath}}) = -\frac{(\dot{y}\,\hat{\boldsymbol{\jmath}})}{2} \quad \longrightarrow \quad \dot{x} = \frac{\dot{y}}{2}.$$

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Rolling

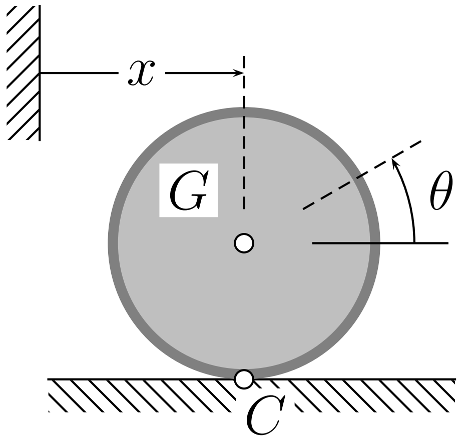
For rolling contact between objects *no slip* implies that the contact point on each object has the same velocity.

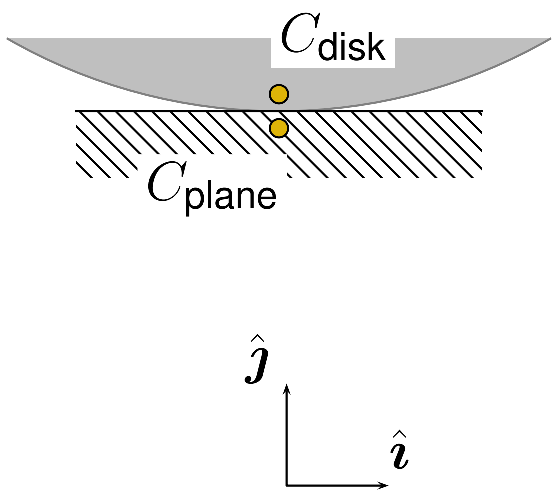
$$\underline{\mathbf{v}}_G = \underline{\mathbf{v}}_{C_{\text{disk}}} + \underline{\boldsymbol{\omega}}_{\mathcal{D}} \times \underline{\mathbf{r}}_{G/C_{\text{disk}}},$$

(rolling on the ground) $\underline{\mathbf{v}}_{C_{\text{disk}}} = \underline{\mathbf{v}}_{C_{\text{plane}}} \equiv \underline{\mathbf{0}}$

$$\underline{\mathbf{v}}_G = \underline{\mathbf{0}} + \underline{\boldsymbol{\omega}}_{\mathcal{D}} \times \underline{\mathbf{r}}_{G/C_{\text{disk}}},$$

$$\dot{x}\,\hat{\boldsymbol{\imath}} = \left(\dot{\theta}\,\hat{\boldsymbol{k}}\right) \times (r\,\hat{\boldsymbol{\jmath}}) = -r\,\dot{\theta}\,\hat{\boldsymbol{\imath}}, \quad \longrightarrow \quad \dot{x} = -r\,\dot{\theta}.$$





Integrating

$$\int_0^t \left\{ \dot{x} = -r\,\dot{\theta} \right\} dt,$$

$$x(t) - x(0) = -\left(r\,\theta(t) - r\,\theta(0)\right),$$

$$x(t) = -r\,\theta(t) + \left(x(0) + r\,\theta(0)\right).$$

If x and θ are measured from a common configuration ($x = 0$ when $\theta = 0$), then $x(0) + r\,\theta(0) = 0$, and

$$x(t) = -r\,\theta(t).$$

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