Coordinates

Engineering Mechanics: Dynamics

D. Dane Quinn, PhD

Department of Mechanical Engineering
The University of Akron
Akron OH 44325–3903 USA

quinn@uakron.edu
www.uakron.edu

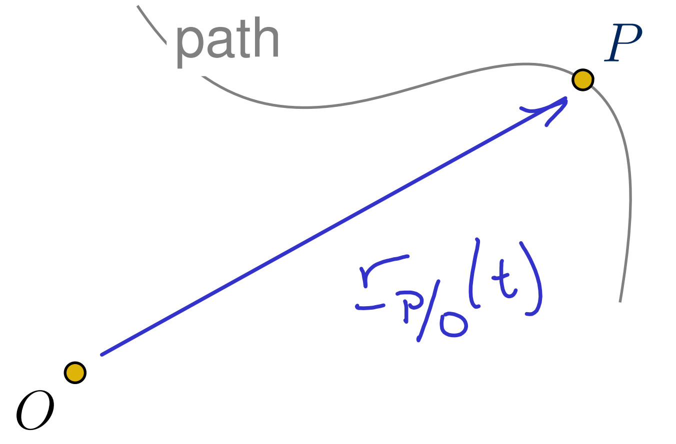
Dynamics describes the changing configuration of a system $\longrightarrow$ The configuration is defined in terms of position vectors

WHAT IS NECESSARY TO DESCRIBE POSITION

BASIS - REFERENCE DIRECTIONS

COORDINATES - MEASURABLE QUANTITIES (DISTANCE, ANGLE) THAT SPECIFY THE CONFIGURATION OF THE SYSTEM

- COMPONENTS IN SPECIFIED DIRECTIONS
- DIRECTION WITH RESPECT TO A REFERENCE BASIS



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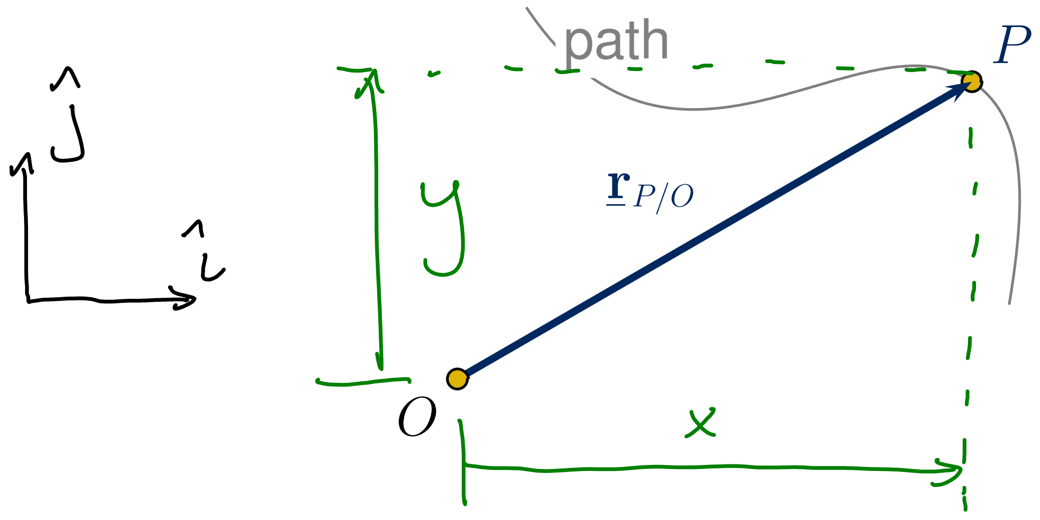
Cartesian Coordinates (René Descartes, 1596–1650)

POSITION VECTORS IN TERMS OF DIRECTIONS

FIXED IN THE GROUND



$$\underline{r}_{P/O} = x\hat{\imath} + y\hat{\jmath}$$

$$\underline{v}_P = \frac{d}{dt}(\underline{r}_{P/O}) = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$$

$$\underline{a}_P = \frac{d}{dt}(\underline{v}_P) = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

IF $\ddot{x} = f(t) \quad (y \equiv 0)$

$$\int_0^t \left\{ \ddot{x}(\xi) = f(\xi) \right\} d\xi \longrightarrow \dot{x}(t) - \dot{x}(0) = \int_0^t f(\xi)\,d\xi$$

$$\int_0^t \left\{ \dot{x}(\tau) = \dot{x}(0) + \int_0^\tau f(\xi)\,d\xi \right\} d\tau \longrightarrow x(t) - x(0) = \dot{x}(0)t + \int_0^t \int_0^\tau f(\xi)\,d\xi\,d\tau$$

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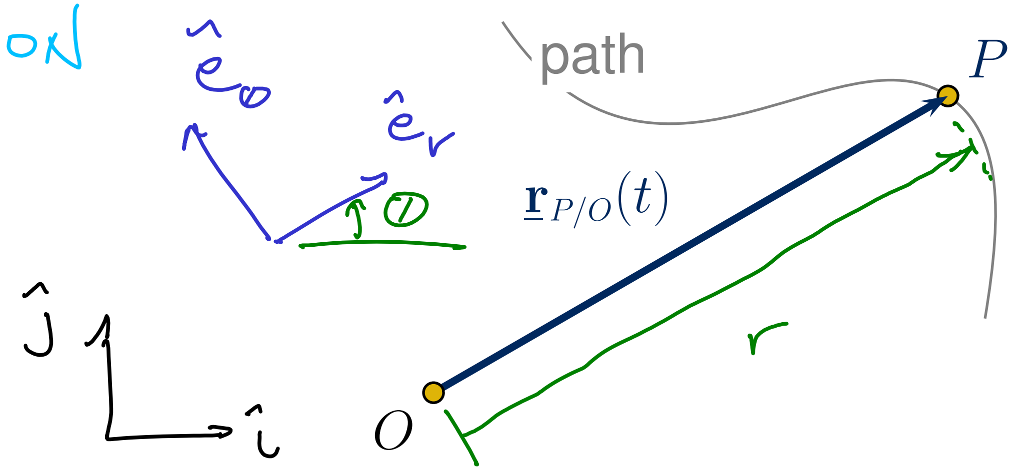
Polar Coordinates

POSITION VECTOR IS DESCRIBED IN TERMS OF DIRECTIONS THAT MOVE WITH THE POSITION

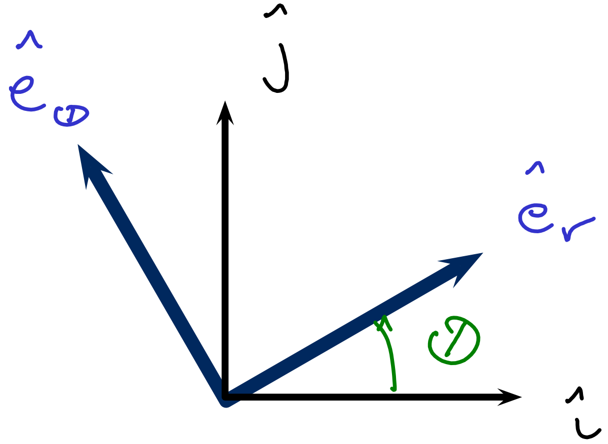
$$\underline{r}_{P/O} = r\,\hat{e}_r$$



THE DIRECTIONS $(\hat{e}_r, \hat{e}_\theta)$ SPIN WITH ANGULAR VELOCITY $\underline{\omega} = \dot{\theta}\hat{k}$

$$\frac{d}{dt}(\hat{e}_r) = \underline{\omega} \times \hat{e}_r = \dot{\theta}\hat{k} \times \hat{e}_r = \dot{\theta}\hat{e}_\theta$$

$$\frac{d}{dt}(\hat{e}_\theta) = \underline{\omega} \times \hat{e}_\theta = \dot{\theta}\hat{k} \times \hat{e}_\theta = -\dot{\theta}\hat{e}_r$$



$$\hat{e}_r = C_\theta \hat{i} + S_\theta \hat{j}$$

$$\hat{e}_\theta = -S_\theta \hat{i} + C_\theta \hat{j}$$

$$\hat{i} = C_\theta \hat{e}_r - S_\theta \hat{e}_\theta$$

$$\hat{j} = S_\theta \hat{e}_r + C_\theta \hat{e}_\theta$$

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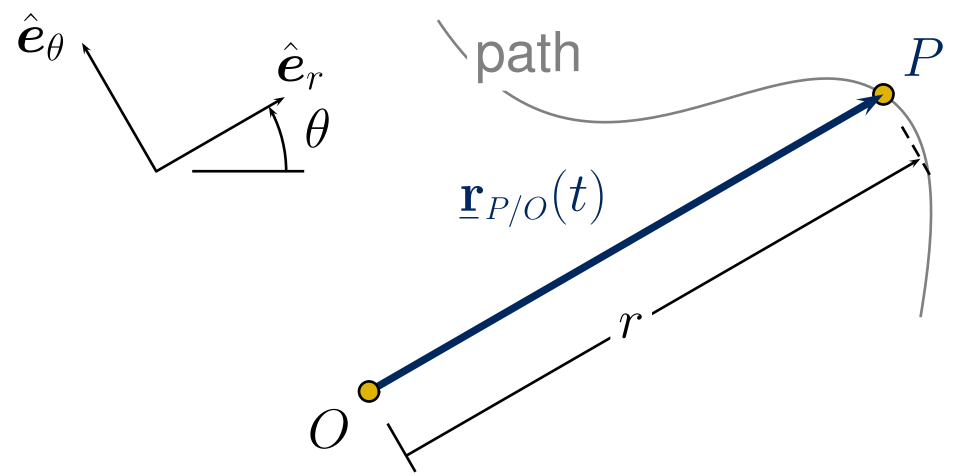
$\underline{r}_{P/O} = r\hat{e}_r$

VELOCITY

$$\underline{v}_P = \frac{d}{dt}\left(\underline{r}_{P/O}\right) = \frac{dr}{dt}\hat{e}_r + r\frac{d}{dt}\left(\hat{e}_r\right)$$

$$= \underbrace{\dot{r}\hat{e}_r}_{\text{RADIAL}} + \underbrace{r\dot{\theta}\hat{e}_\theta}_{\text{TRANSVERSE}}$$



&

ACCELERATION

$$\underline{a}_P = \frac{d}{dt}\left(\underline{v}_P\right) = \frac{d}{dt}\left(\dot{r}\right)\hat{e}_r + \dot{r}\underbrace{\frac{d}{dt}\left(\hat{e}_r\right)}_{\dot{\theta}\hat{e}_\theta}$$

$$+ \frac{d}{dt}\left(r\dot{\theta}\right)\hat{e}_\theta + r\dot{\theta}\underbrace{\frac{d}{dt}\left(\hat{e}_\theta\right)}_{-\dot{\theta}\hat{e}_r}$$

$$= \underbrace{\left(\ddot{r} - r\dot{\theta}^2\right)}_{\text{RADIAL}}\hat{e}_r + \underbrace{\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)}_{\text{TRANSVERSE}}\hat{e}_\theta$$

$$\frac{d}{dt}\left(r\dot{\theta}\right) = \dot{r}\dot{\theta} + r\ddot{\theta}$$

SPEED

$$v = \|\underline{v}_P\| = \sqrt{\underline{v}_P \cdot \underline{v}_P}$$

$$= \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$= \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$$

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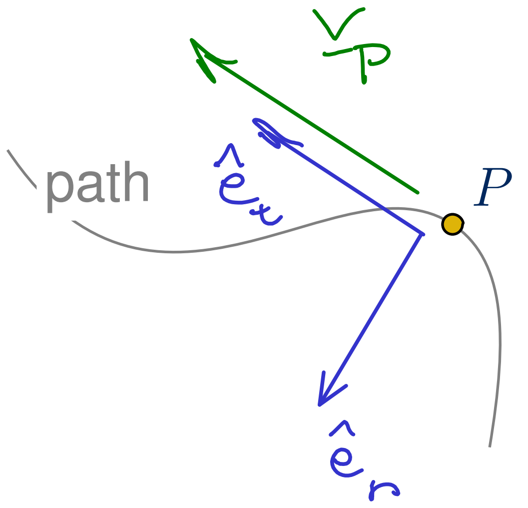
Normal and Tangential Coordinates

VELOCITY CAN BE DEFINED AS

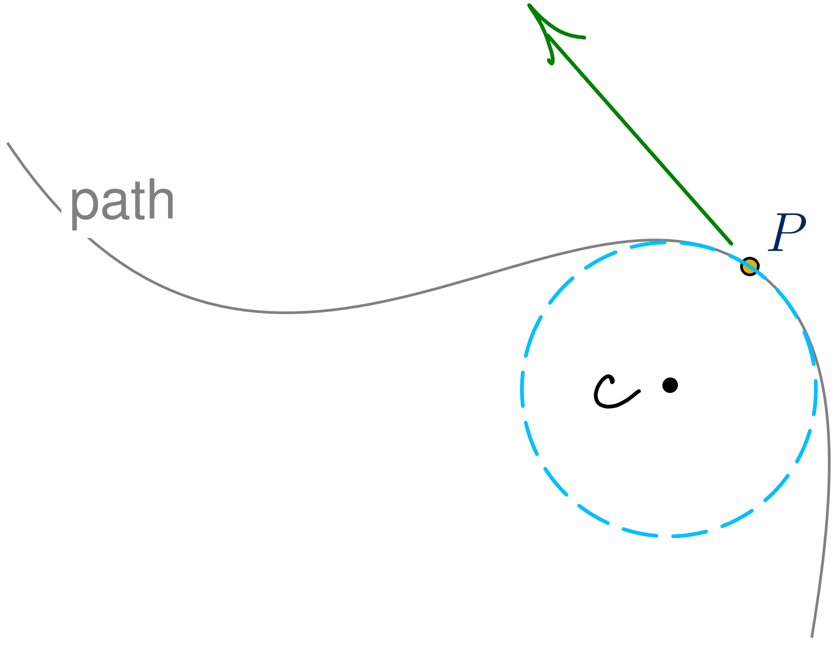
$\underline{v}_P = v\,\hat{e}_t$



DIRECTIONS $(\hat{e}_t, \hat{e}_n)$ MOVE WITH THE VELOCITY

& THE ACCELERATION IS

$\underline{a}_P = \dot{v}\,\hat{e}_t + \dfrac{v^2}{\rho}\,\hat{e}_n$



ρ: RADIUS OF CURVATURE

FOR A CIRCULAR PATH $\rho \equiv r$ (RADIUS)

IF $y(x) = f(x)$

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

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