Particle Dynamics

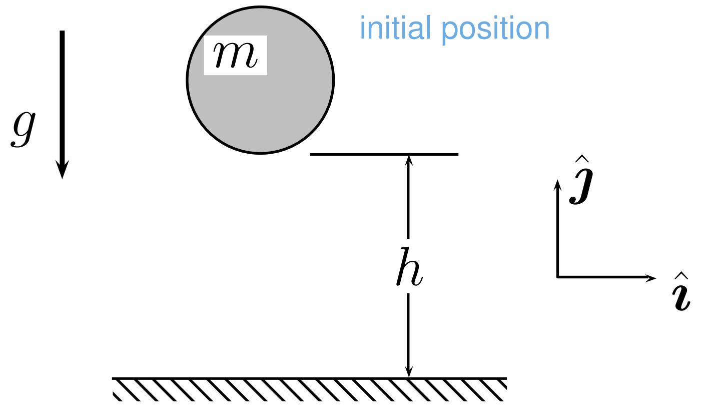
Engineering Mechanics: Dynamics

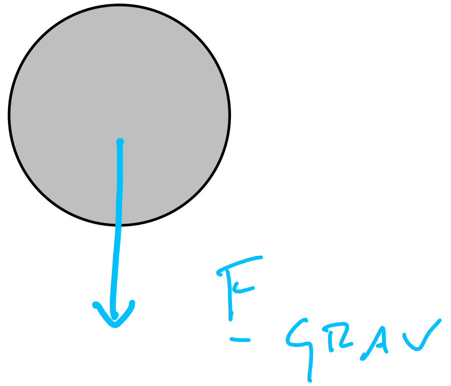
D. Dane Quinn, PhD

Department of Mechanical Engineering
The University of Akron
Akron OH 44325–3903 USA

quinn@uakron.edu
www.uakron.edu

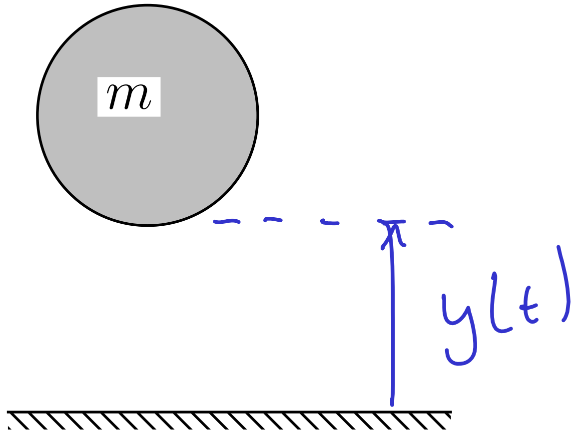
A ball of mass m is released from rest at an initial height h above the ground. Determine the time required for the ball to hit the ground, and its speed when the impact occurs.





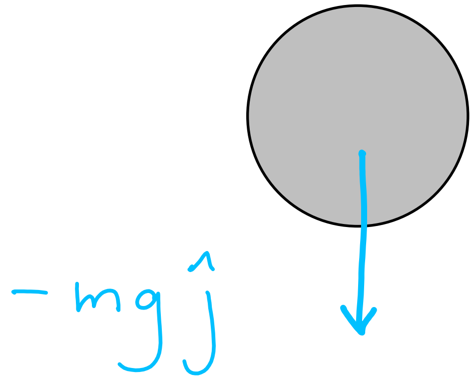
ONLY FORCE ARISES FROM GRAVITY

Coordinates and Directions/Kinematics/Free Body Diagram



$y(0) = h$

$\underline{a}_B(t) = \ddot{y}(t)\,\hat{j}$



quinn@uakron.edu
www.uakron.edu

Equations of Motion

LINEAR MOMENTUM BALANCE

$$-mg\hat{j} = \sum \underline{F} = m\underline{a}_B = m(\ddot{y}\hat{j}) \longrightarrow \ddot{y} = -g$$

INTEGRATE TO FIND VELOCITY

$$\int_0^t \left\{ \ddot{y}(\tau) = -g \right\} d\tau \longrightarrow \dot{y}(t) - \dot{y}(0) = -gt \qquad (\dot{y}(0) = 0)$$

$$\dot{y}(t) = -gt$$

INTEGRATE TO FIND POSITION

$$\int_0^t \left\{ \dot{y}(\tau) = -g\tau \right\} d\tau \longrightarrow y(t) - y(0) = -\frac{g}{2}t^2 \qquad (y(0) = h)$$

$$y(t) = -\frac{g}{2}t^2 + h$$

quinn@uakron.edu
www.uakron.edu

CONTACT OCCURS AT $t = t_f$, & $y(t_f) = 0$

FINAL CONDITION

$$\boxed{0 = \boxed{y(t_f) = -\frac{g}{2}t_f^2 + h}}$$

RESPONSE

& SO

$$t_f = \sqrt{\frac{2h}{g}}$$

THE VELOCITY IS $\underline{v}_B = \dot{y}\,\hat{j}$

$$\dot{y}(t_f) = -g t_f = -\sqrt{2gh}$$

quinn@uakron.edu
www.uakron.edu