Particle Dynamics

Engineering Mechanics: Dynamics

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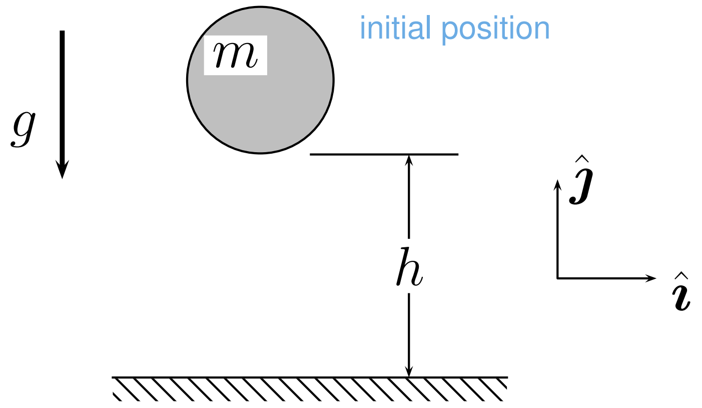
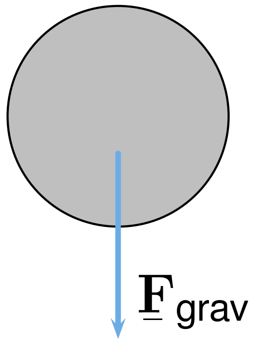
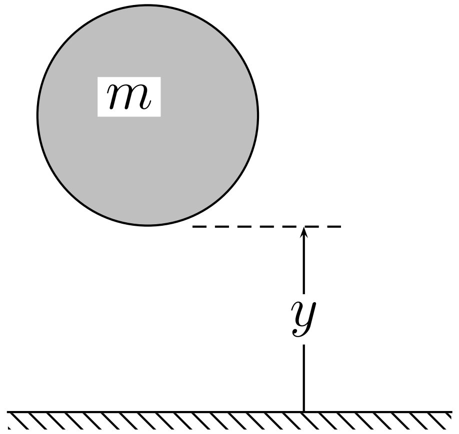
A ball of mass m is released from rest at an initial height h above the ground. Determine the time required for the ball to hit the ground, and its speed when the impact occurs.





Only the gravitational force acts on the particle, while the motion of the particle is only in the $\hat{\jmath}$ direction.

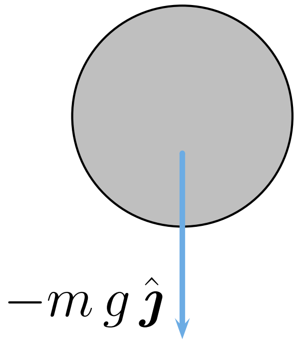
Coordinates and Directions/Kinematics/Free Body Diagram



We measure the displacement of the ball above the ground as y, with $y(0) = h$, so that the acceleration of the ball is

$$\underline{\mathbf{a}}_B = \ddot{y}\,\hat{\jmath}.$$

The free body diagram is shown to the right.



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Equations of Motion

Finally, applying linear momentum balance to this particle

$$\sum \underline{\mathbf{F}} = -m\,g\,\hat{\boldsymbol{\jmath}} = m\,\ddot{y}\,\hat{\boldsymbol{\jmath}} = m\,\underline{\mathbf{a}}_B, \quad \longrightarrow \quad \ddot{y} = -g, \quad \text{(constant acceleration)}$$

Integrating to find the velocity

$$\int_0^t \left\{ \ddot{y}(\tau) = -g \right\} d\tau \qquad \longrightarrow \qquad \dot{y}(t) - \dot{y}(0) = -g\,t, \qquad (\dot{y}(0) = 0)$$

$$\dot{y}(t) = -g\,t,$$

and the position

$$\int_0^t \left\{ \dot{y}(\tau) = -g\,\tau \right\} d\tau \qquad \longrightarrow \qquad y(t) - y(0) = -\frac{g\,t^2}{2}, \qquad (y(0) = h)$$

$$y(t) = h - \frac{g\,t^2}{2},$$

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When the ball impacts the surface at time $t = t_f$, the vertical displacment vanishes $y(t_f) = 0$, so that

$$\overbrace{0 = y(t_f)}^{\text{final condition}} \underbrace{= h - \frac{g\,t_f^2}{2}}_{\text{response}}, \quad \longrightarrow \quad t_f = \sqrt{\frac{2\,h}{g}},$$

while the speed at this instant is

$$\dot{y}(t_f) = -g\,t_f = \sqrt{2\,g\,h}.$$

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