Particle Dynamics

Engineering Mechanics: Dynamics

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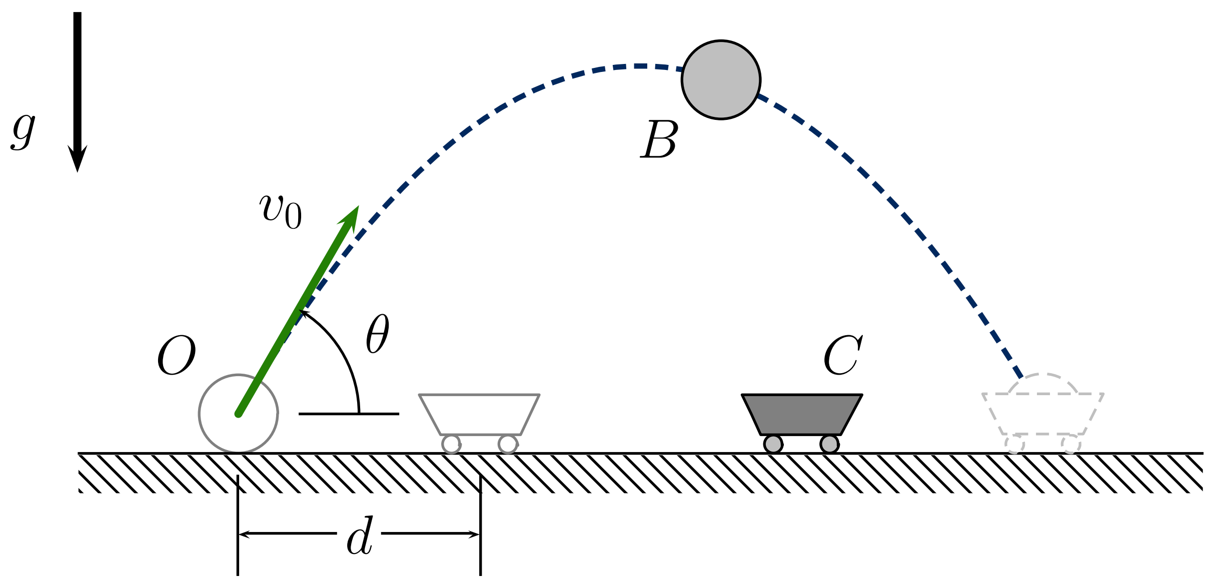
A particle of mass m is launched with an initial inclination θ while a cart moves with constant velocity $\underline{\mathbf{v}}_C = u\,\hat{\boldsymbol{\imath}}$ and is initially located a distance d from the particle. Find the initial speed of the mass so that it lands in the cart. How far must the ball be thrown?

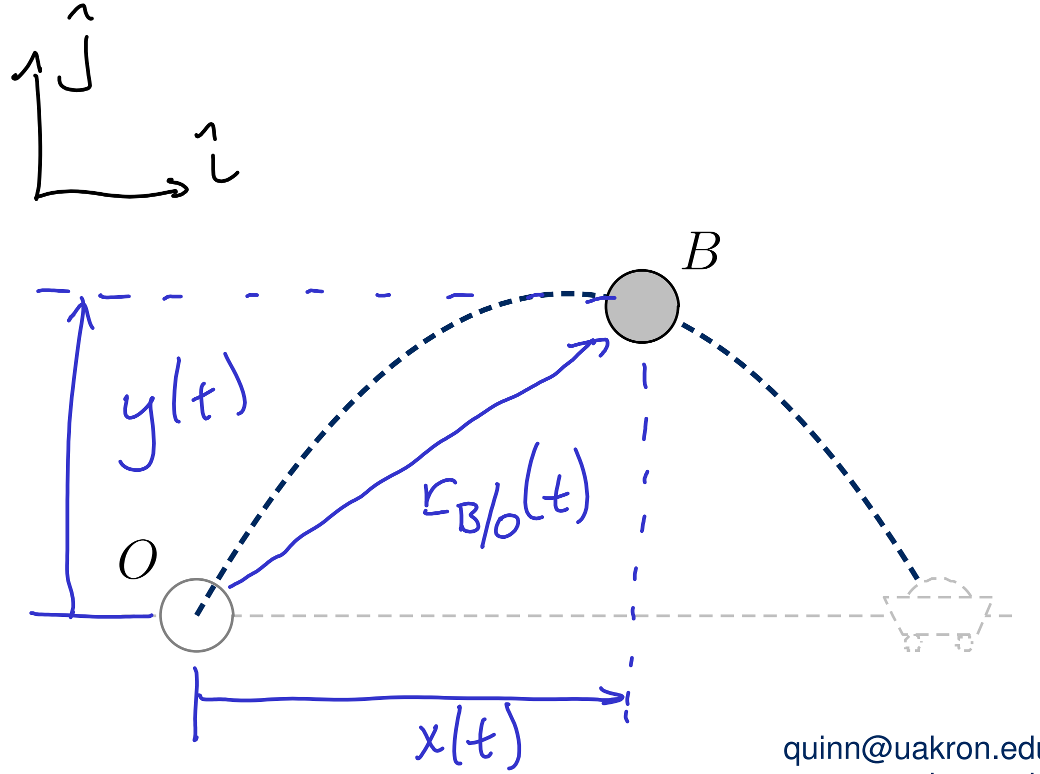


Coordinates and Directions

ONLY THE GRAVITIONAL FORCES

ASSUME PLANAR MOTION

$$r_{B/O}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$



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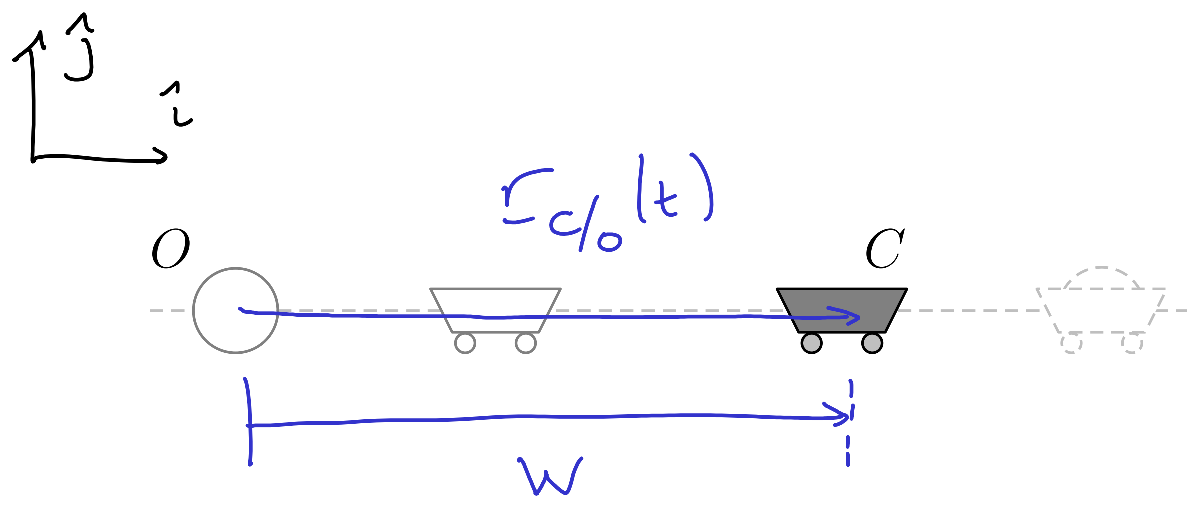
POSITION OF C

$\underline{r}_{c/o}(t) = w(t)\,\hat{\imath}\;;\qquad \underline{v}_c = \dot{w}\,\hat{\imath} = u\,\hat{\imath}$



$\int_0^t \left\{ \dot{w}(\tau) = u \right\} d\tau \longrightarrow w(t) - w(0) = ut$

$w(0) = d \qquad w(t) = ut + d$

Equations of Motion

LINEAR MOMENTUM BALANCE

$\ddot{x} = 0$

$\ddot{y} = -g$

$\underline{v}_B(0) = v_x\,\hat{\imath} + v_y\,\hat{j} = v_0\left(C_\theta\,\hat{\imath} + S_\theta\,\hat{j}\right)$

$v_x = v_0 C_\theta\,,\quad v_y = v_0 S_\theta$

$x_0 = 0 \qquad y_0 = 0$

SO THAT

$\dot{x}(t) = v_0 C_\theta \qquad \dot{y}(t) = -gt + v_0 S_\theta$

$x(t) = v_0 C_\theta \cdot t \qquad y(t) = -\frac{g}{2}t^2 + v_0 S_\theta \cdot t$

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AT TIME $t = t_f$ THE BALL LANDS IN THE CART

$$\underline{r}_{B/O}(t_f) = \underline{r}_{C/O}(t_f) \longrightarrow x(t_f)\,\hat{\imath} + y(t_f)\,\hat{\jmath} = w(t_f)\,\hat{\imath} + 0\,\hat{\jmath}$$

$\hat{\imath}$ DIR: $x(t_f) = w(t_f)$

$$\boxed{v_0 C_\theta t_f = d + u\,t_f}$$

$\hat{\jmath}$ DIR: $y(t_f) = 0$

$$\boxed{-\frac{g}{2} t_f^2 + v_0 S_\theta t_f = 0}$$

SOLVE FOR t_f

$$t_f\left(-\frac{g}{2} t_f + v_0 S_\theta\right) = 0 \qquad t_f = 0 \qquad t_f = \frac{2S_\theta}{g} v_0$$

SO THAT

$$\left(v_0 C_\theta - u\right)\frac{2S_\theta}{g} v_0 = d \longrightarrow \frac{2S_\theta C_\theta}{g} v_0^2 - \frac{2uS_\theta}{g} v_0 - d = 0$$

SOLVE

$$v_0 = \frac{u}{2C_\theta} \pm \sqrt{\left(\frac{u}{2C_\theta}\right)^2 + \frac{dg}{2S_\theta C_\theta}} = \frac{u}{2C_\theta}\left\{1 \pm \sqrt{1 + \frac{2dgC_\theta}{u^2 S_\theta}}\right\}$$

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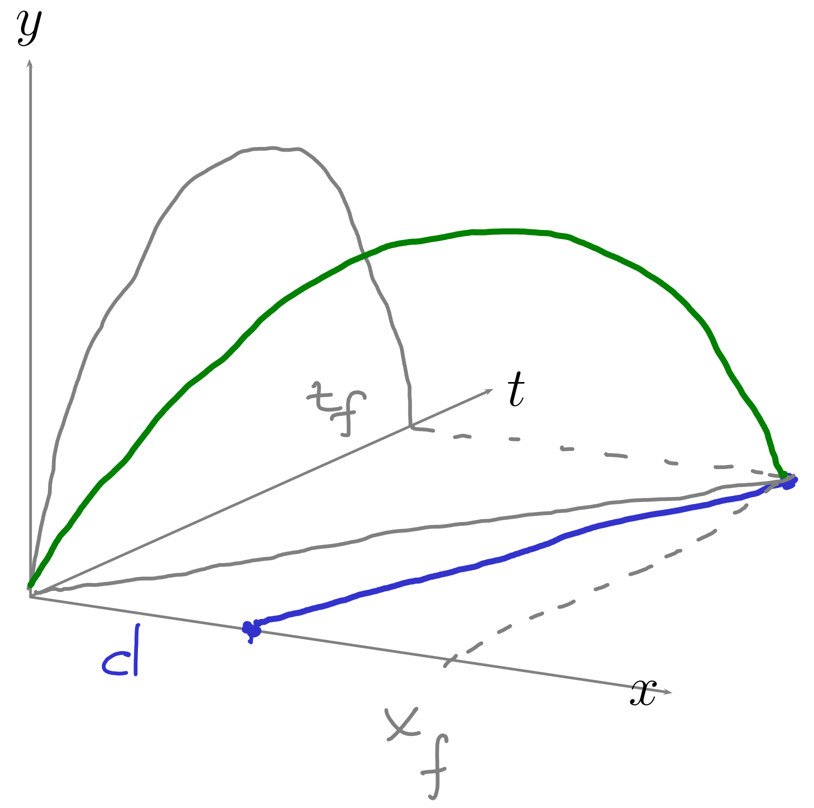
THERE ARE TWO SOLUTIONS

$$v_0 = \frac{u}{2C_\phi}\left\{1+\sqrt{1+\frac{2dgC_\phi}{u^2S_\phi}}\right\}$$

& SO

$$t_f = \frac{2v_0S_\phi}{g} = \frac{uT_\phi}{g}\left\{1+\sqrt{1+\frac{2dgC_\phi}{u^2S_\phi}}\right\}$$



& FINALLY

$$x_f = x(t_f) = w(t_f)$$
$$= d + \frac{u^2T_\phi}{g}\left\{1+\sqrt{1+\frac{2dgC_\phi}{u^2S_\phi}}\right\}$$

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