Particle Dynamics

Engineering Mechanics: Dynamics

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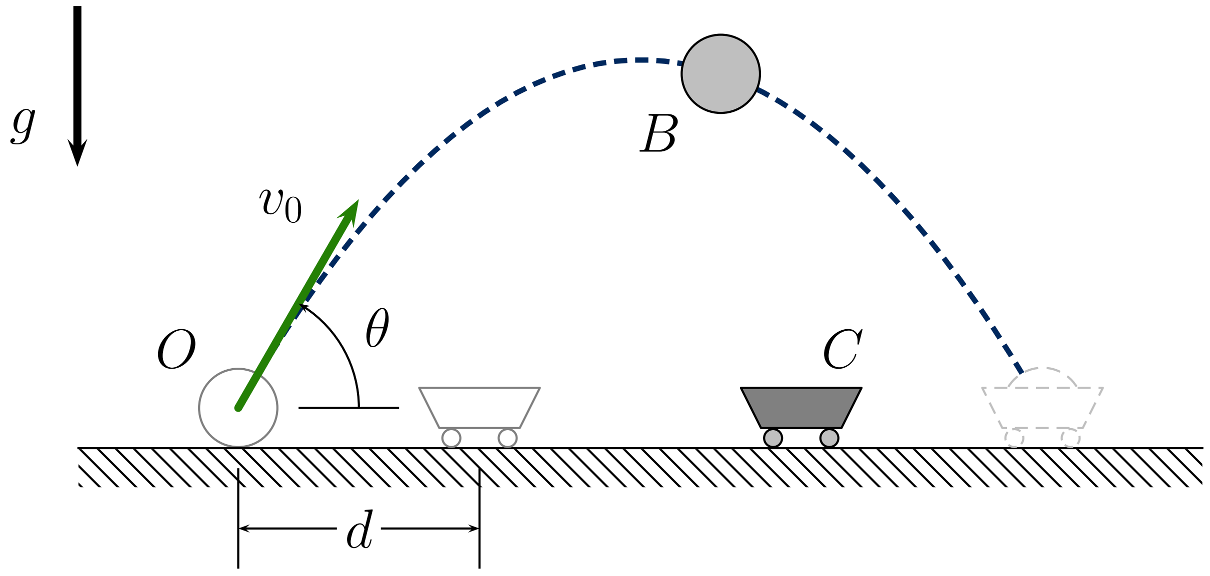
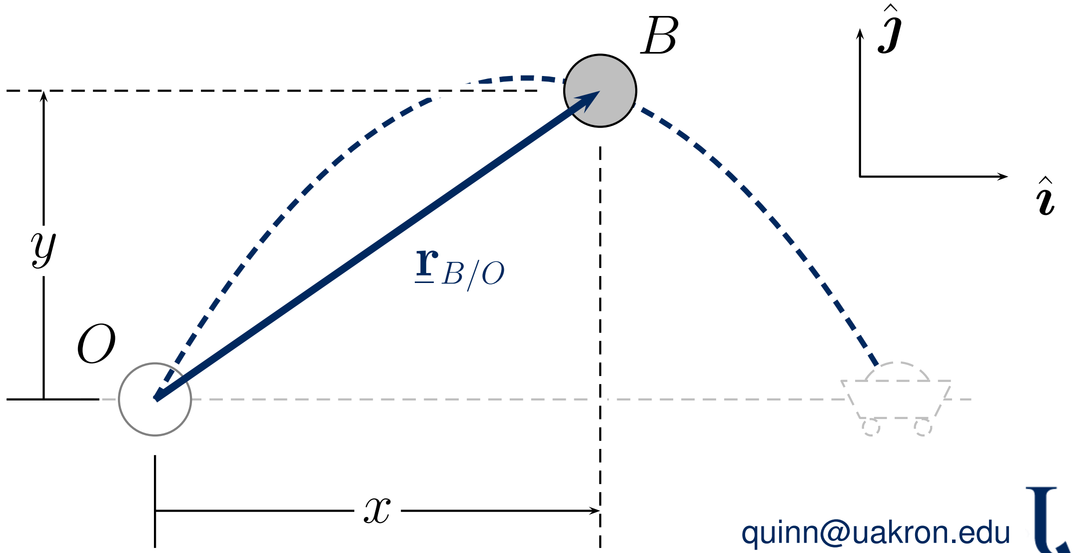
A particle of mass m is launched with an initial inclination θ while a cart moves with constant velocity $\underline{\mathbf{v}}_C = u\,\hat{\boldsymbol{\imath}}$ and is initially located a distance d from the particle. Find the initial speed of the mass so that it lands in the cart. How far must the ball be thrown?



Coordinates and Directions

Only the gravitational force acts on the particle, while the motion of the particle is in the plane, so that we define x and y to describe the displacement of the ball as

$$\underline{\mathbf{r}}_{B/O}(t) = x(t)\,\hat{\boldsymbol{\imath}} + y(t)\,\hat{\boldsymbol{\jmath}}.$$



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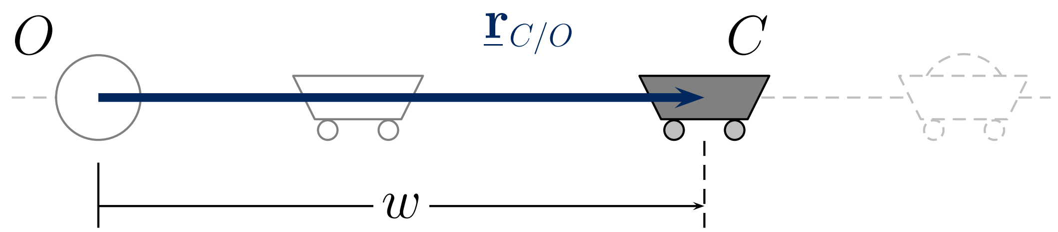
The displacement of the cart is defined as

$$\underline{\mathbf{r}}_{C/O} = w\,\hat{\boldsymbol{\imath}}, \quad \underline{\mathbf{v}}_C = \dot{w}\,\hat{\boldsymbol{\imath}} = u\,\hat{\boldsymbol{\imath}},$$



so that

$$\int_0^t \left\{ \dot{w}(\tau) = u \right\} d\tau, \quad \longrightarrow \quad w(t) - w(0) = u\,t, \quad (w(0) \equiv d)$$

$$w(t) = u\,t + d.$$

Equations of Motion

The equations of motion are

$$\ddot{x} = 0, \qquad \ddot{y} = -g, \qquad \text{with} \quad \begin{array}{ll} v_x = v_0\,\mathrm{C}_\theta, & v_y = v_0\,\mathrm{S}_\theta, \\ x_0 = 0, & y_0 = 0, \end{array}$$

so that

$$\dot{x}(t) = v_0\,\mathrm{C}_\theta, \qquad \dot{y}(t) = -g\,t + v_0\,\mathrm{S}_\theta,$$

$$x(t) = v_0\,\mathrm{C}_\theta\,t, \qquad y(t) = -\frac{g\,t^2}{2} + v_0\,\mathrm{S}_\theta\,t.$$

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At time $t = t_f$ when the ball lands in the cart

$$\underline{\mathbf{r}}_{B/O}(t_f) = \underline{\mathbf{r}}_{C/O}(t_f), \quad \longrightarrow \quad x(t_f)\,\hat{\boldsymbol{\imath}} + y(t_f)\,\hat{\boldsymbol{\jmath}} = w(t_f)\,\hat{\boldsymbol{\imath}}$$

so that

$$x(t_f) = w(t_f), \qquad y(t_f) = 0,$$

$$v_0\,\mathrm{C}_\theta\,t_f = d + u\,t_f, \qquad -\frac{g\,{t_f}^2}{2} + v_0\,\mathrm{S}_\theta\,t_f = 0.$$

From these, we can solve for t_f as

$$t_f = \frac{2\,v_0\,\mathrm{S}_\theta}{g},$$

so that

$$(v_0\,\mathrm{C}_\theta - u)\,\frac{2\,v_0\,\mathrm{S}_\theta}{g} = d, \quad \longrightarrow \quad (\mathrm{C}_\theta)\;{v_0}^2 - (u)\;v_0 - \frac{d\,g}{2\,\mathrm{S}_\theta} = 0.$$

Finally solving this quadratic equation for the initial speed yields

$$v_0 = \frac{u}{2\,\mathrm{C}_\theta} \pm \sqrt{\left(\frac{u}{2\,\mathrm{C}_\theta}\right)^2 + \frac{d\,g}{2\,\mathrm{S}_\theta\,\mathrm{C}_\theta}} = \frac{u}{2\,\mathrm{C}_\theta}\left\{1 \pm \sqrt{1 + \frac{2\,d\,g\,\mathrm{C}_\theta}{u^2\,\mathrm{S}_\theta}}\right\}$$

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Note that there are two solutions for v_0, and only one of them is positive, so that

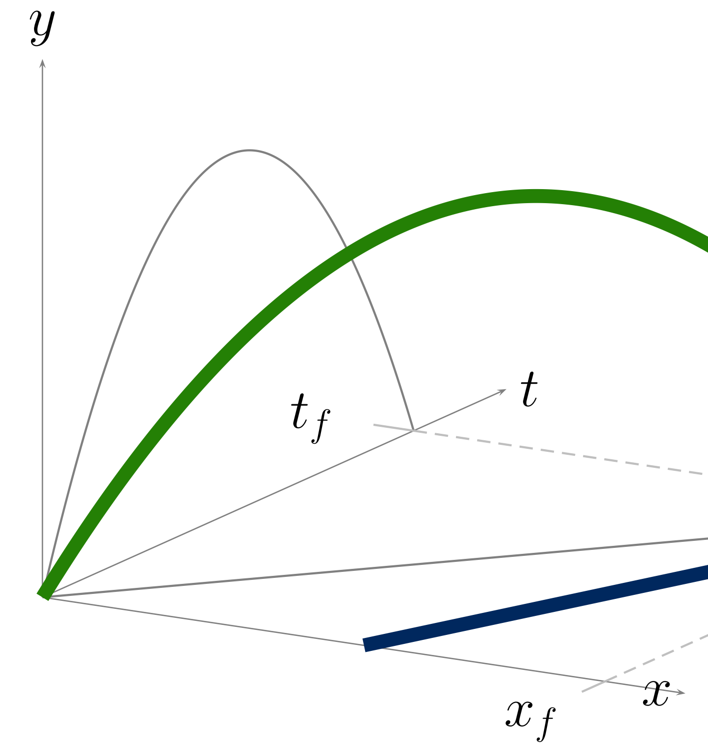
$$v_0 = \frac{u}{2\,\mathrm{C}_\theta}\left\{1 + \sqrt{1 + \frac{2\,d\,g\,\mathrm{C}_\theta}{u^2\,\mathrm{S}_\theta}}\right\}.$$

Therefore, the time and distance at which the mass lands in the cart is

$$t_f = \frac{2\,v_0\,\mathrm{S}_\theta}{g} = \frac{u\,\mathrm{T}_\theta}{g}\left\{1 + \sqrt{1 + \frac{2\,d\,g\,\mathrm{C}_\theta}{u^2\,\mathrm{S}_\theta}}\right\},$$

and finally

$$\begin{aligned} x_f &= x(t_f) = w(t_f), \\ &= d + \frac{u^2\,\mathrm{T}_\theta}{g}\left\{1 + \sqrt{1 + \frac{2\,d\,g\,\mathrm{C}_\theta}{u^2\,\mathrm{S}_\theta}}\right\} \end{aligned}$$



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