Particle Dynamics

Engineering Mechanics: Dynamics

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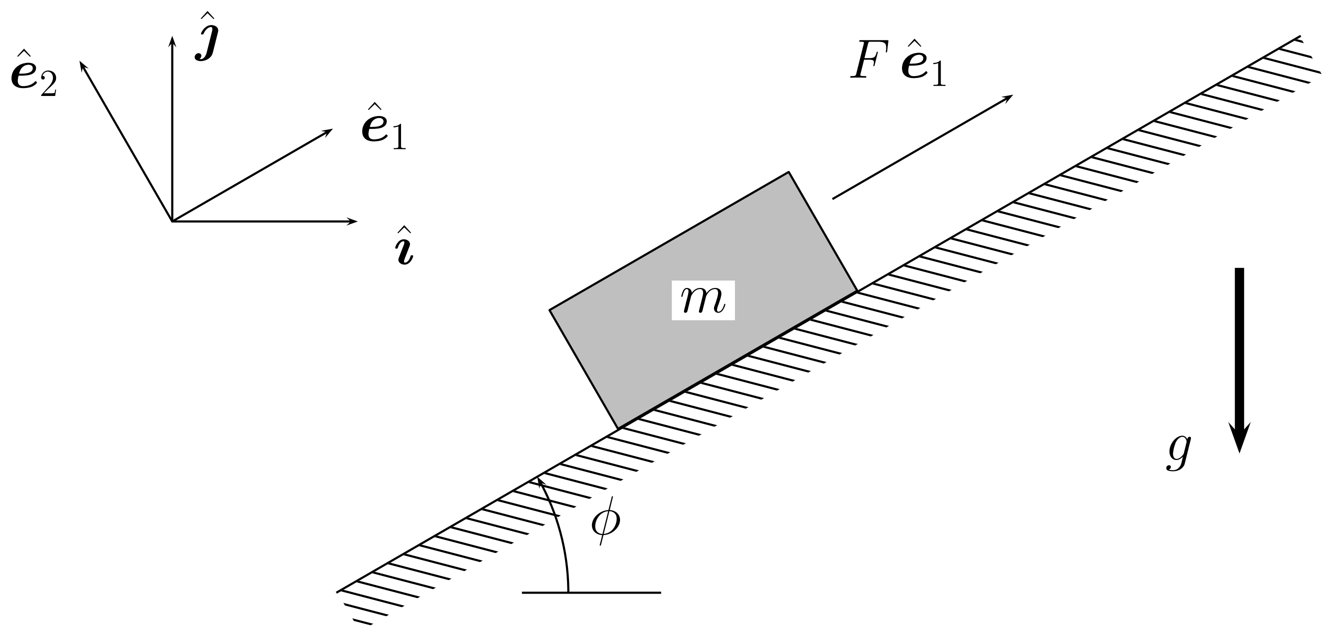
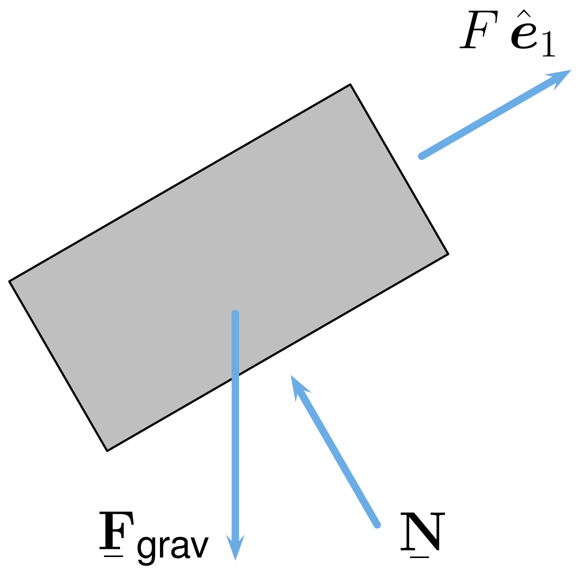
A block of mass m slides on a smooth surface inclined at an angle ϕ and is pulled by a force of magnitude F. Find the acceleration of the block.





The forces acting on the block arise from the

- Weight ($\underline{\mathbf{F}}_{\text{grav}}$),
- Normal force ($\underline{\mathbf{N}}$), and
- External force ($F\,\hat{\boldsymbol{e}}_1$),

while the motion of the block is only in the $\hat{e}_1$ direction.

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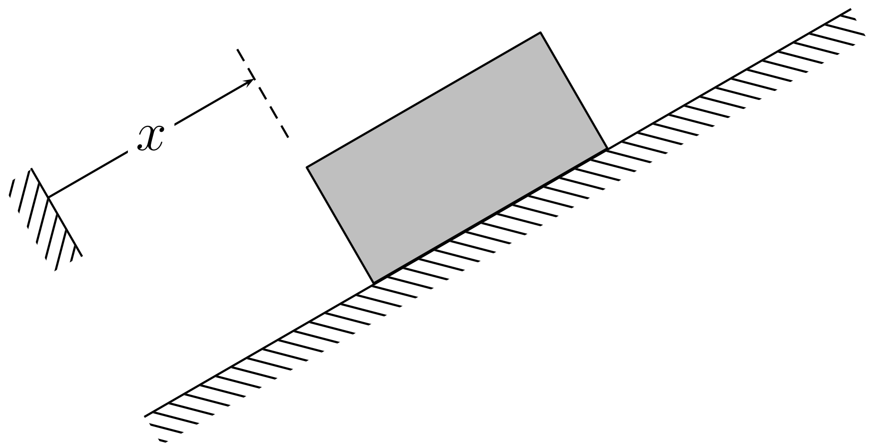
Coordinates and Directions



We measure the displacement of the block as x, and the directions $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}})$ and $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2)$ are related as

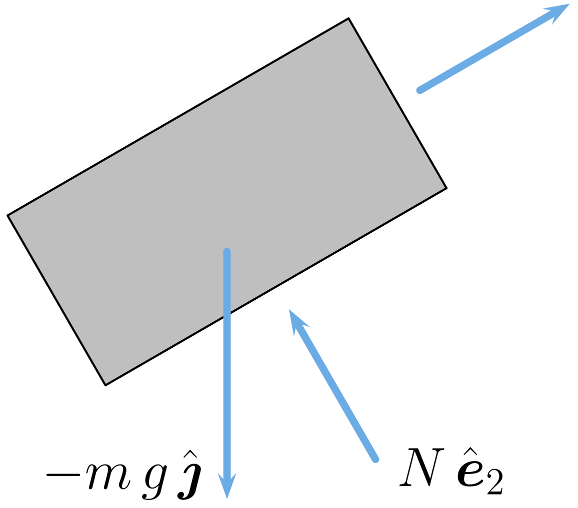
$$\hat{\boldsymbol{\imath}} = \mathrm{C}_\phi \, \hat{\boldsymbol{e}}_1 - \mathrm{S}_\phi \, \hat{\boldsymbol{e}}_2, \qquad \hat{\boldsymbol{\jmath}} = \mathrm{S}_\phi \, \hat{\boldsymbol{e}}_1 + \mathrm{C}_\phi \, \hat{\boldsymbol{e}}_2.$$

Kinematics/Free Body Diagram

With this, the acceleration of the block is

$$\underline{\mathbf{a}}_B = \ddot{x} \, \hat{\boldsymbol{e}}_1,$$

and the free body diagram is shown to the right.



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Equations of Motion

Applying linear momentum balance to the block

$$\sum \underline{\mathbf{F}} = -m\,g\,\hat{\boldsymbol{j}} + F\,\hat{\boldsymbol{e}}_1 + N\,\hat{\boldsymbol{e}}_2 = m\,\ddot{x}\,\hat{\boldsymbol{e}}_1 = m\,\underline{\mathbf{a}}_B.$$

Taking components in the $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2)$ directions

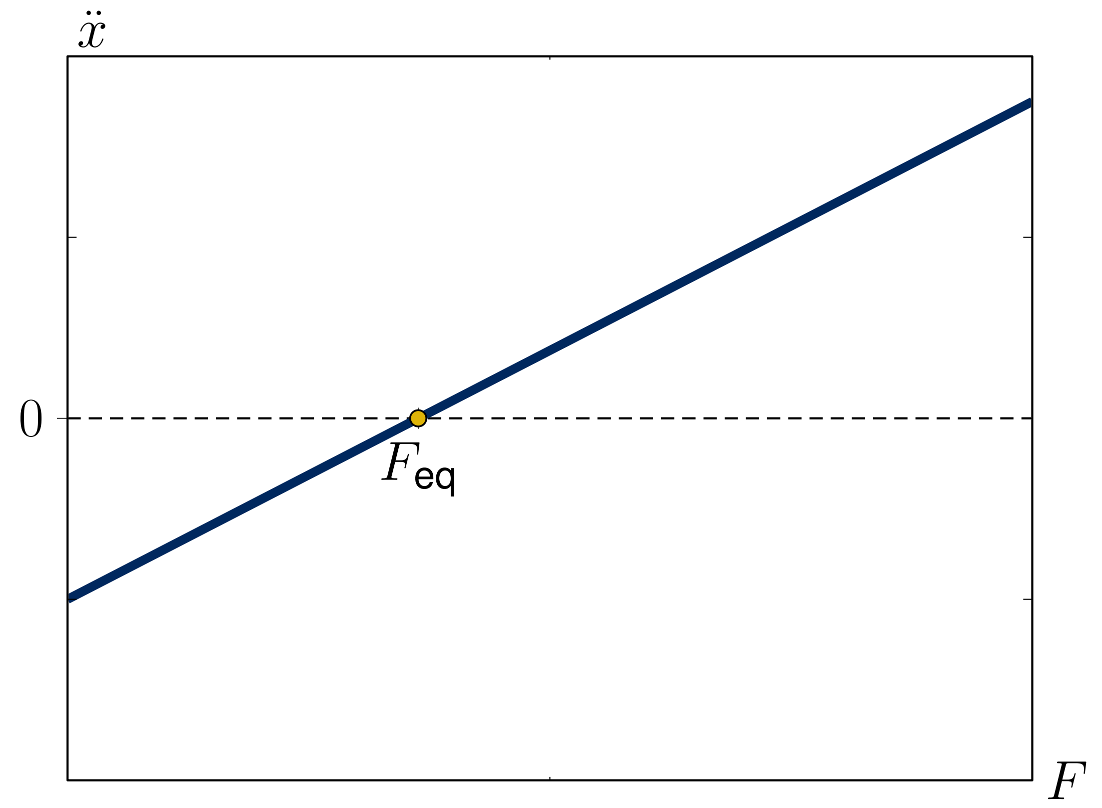
$\hat{\boldsymbol{e}}_1$ direction: $$-m\,g\,\mathrm{S}_\phi + F = m\,\ddot{x},$$

$\hat{\boldsymbol{e}}_2$ direction: $$-m\,g\,\mathrm{C}_\phi + N = 0.$$

Finally, the acceleration and normal load can be solved as

$$\ddot{x} = \frac{F}{m} - g\,\mathrm{S}_\phi, \quad N = m\,g\,\mathrm{C}_\phi,$$

$$\longrightarrow \quad \underline{\mathbf{a}}_B = \ddot{x}\,\hat{\boldsymbol{e}}_1 = \left(\frac{F}{m} - g\,\mathrm{S}_\phi\right)\hat{\boldsymbol{e}}_1$$



Note that when $\ddot{x} = 0$ the system is in static equilibrium, which occurs for

$$F_{\text{eq}} = m\,g\,\mathrm{S}_\phi\,.$$

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