Particle Dynamics

Engineering Mechanics: Dynamics

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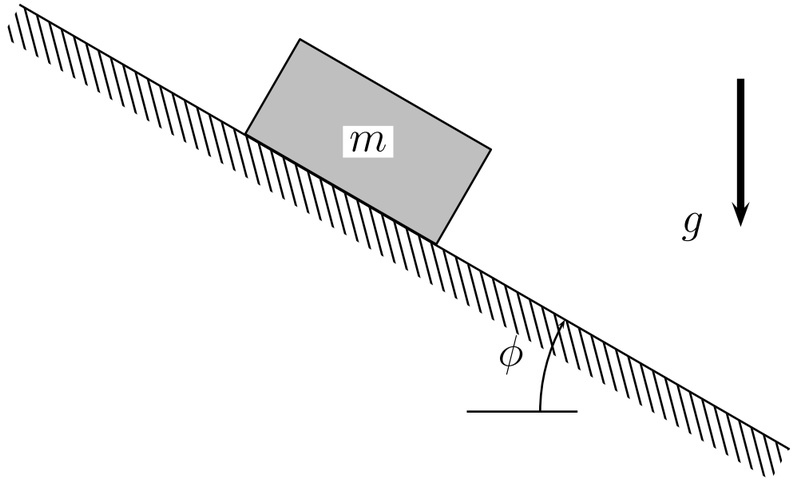
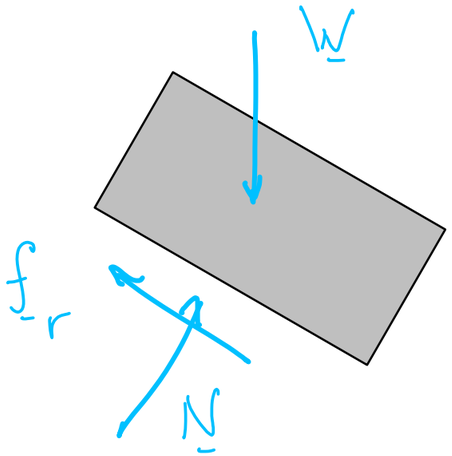
A block of mass m is released from rest and slides on a rough surface with coefficient of friction μ and inclined at an angle ϕ. What is the minimum angle of inclination $\phi = \phi_\star$ necessary for the block to slide? If $\phi > \phi_\star$ find the acceleration of the block.





FORCES

WEIGHT

NORMAL FORCE

FRICTION

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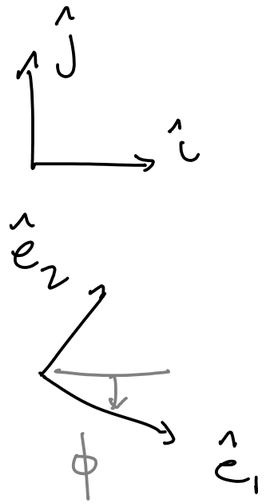
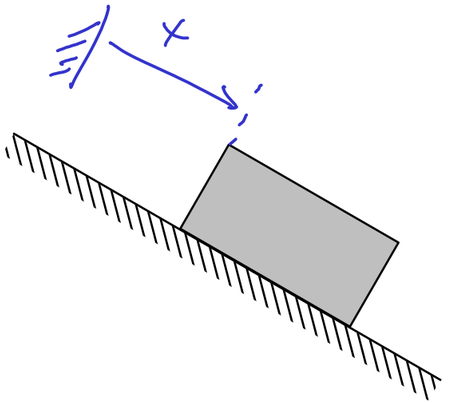
Coordinates and Directions/Kinematics

$$\hat{e}_1 = C_\phi \hat{\imath} - S_\phi \hat{\jmath}$$
$$\hat{e}_2 = S_\phi \hat{\imath} + C_\phi \hat{\jmath}$$

$$\hat{\imath} = C_\phi \hat{e}_1 + S_\phi \hat{e}_2$$
$$\hat{\jmath} = -S_\phi \hat{e}_1 + C_\phi \hat{e}_2$$

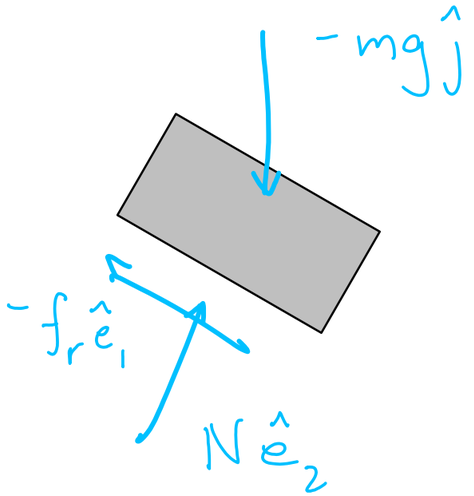




ACCELERATION

$$\underline{a}_B = \ddot{x}\,\hat{e}_1$$

Free Body Diagram/Equations of Motion



LINEAR MOMENTUM BALANCE

$$-mg\hat{\jmath} + N\hat{e}_2 - f_r\hat{e}_1 = \Sigma \underline{F} = m\underline{a}_B = m\ddot{x}\hat{e}_1$$

$$(mgS_\phi - f_r)\hat{e}_1 + (N - mgC_\phi)\hat{e}_2 = m\ddot{x}\hat{e}_1$$

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COMPONENTS

$\hat{e}_1$ DIR $\quad mgS_\phi - f_r = m\ddot{x}$

$\hat{e}_2$ DIR $\quad N - mgC_\phi = 0 \longrightarrow N = mgC_\phi$

$a_B = \ddot{x}\hat{e}_1$

$= g(S_\phi - \mu C_\phi)\hat{e}_1$

STICKS $\quad \ddot{x} = 0$

$mgS_\phi - f_r = 0 \longrightarrow f_r = mgS_\phi$

$|f_r| \le \mu N$

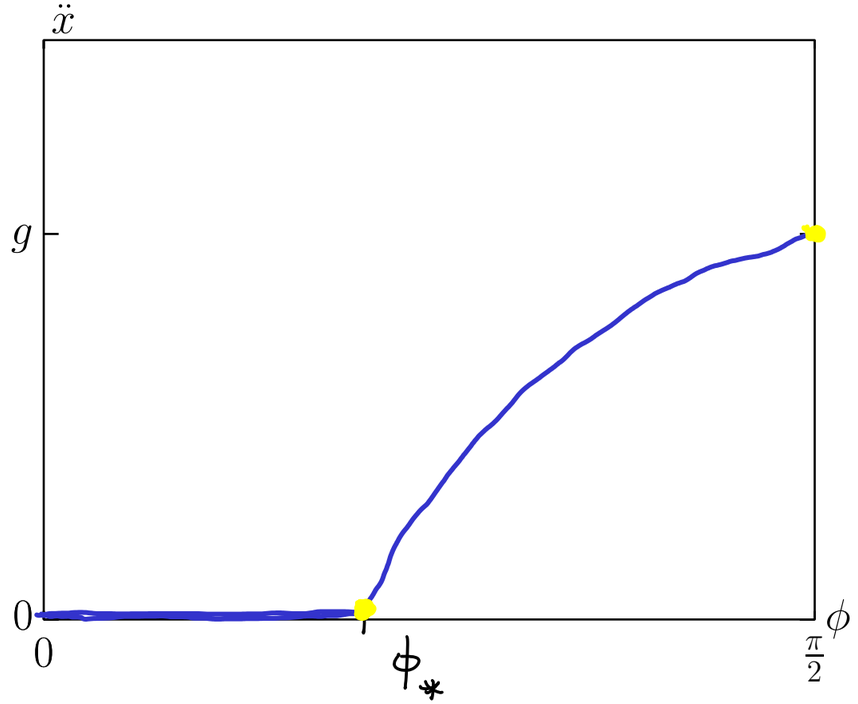
$|mgS_\phi| \le \mu mgC_\phi \longrightarrow T_\phi \le \mu$

CRITICAL ANGLE $\quad \phi_* = T_\mu^{-1}$

IF $\phi > \phi_*$ THEN THE BLOCK SLIDES

$f_r = \mu N$

$m\ddot{x} = mgS_\phi - \mu mgC_\phi \longrightarrow \ddot{x} = g(S_\phi - \mu C_\phi), \quad \phi > \phi_* = T_\mu^{-1}$



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