Particle Dynamics

Engineering Mechanics: Dynamics

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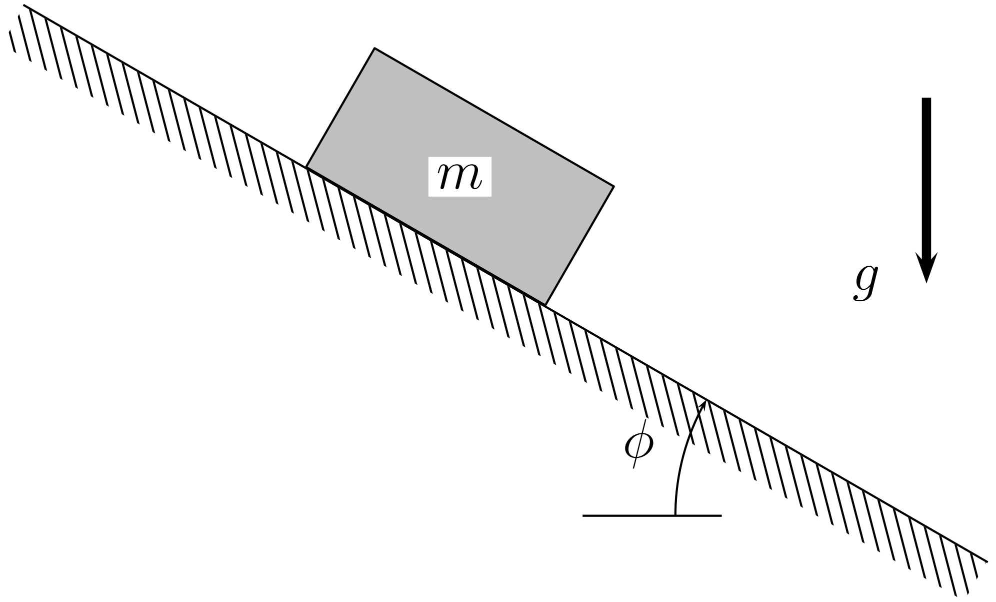
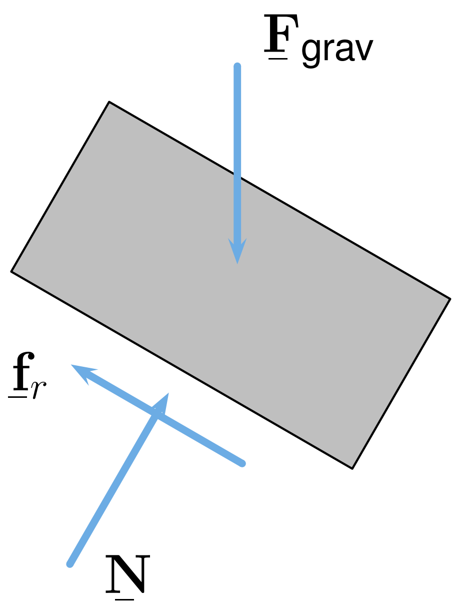
A block of mass m is released from rest and slides on a rough surface with coefficient of friction μ and inclined at an angle ϕ. What is the minimum angle of inclination $\phi = \phi_\star$ necessary for the block to slide? If $\phi > \phi_\star$ find the acceleration of the block.





The forces acting on the block arise from

- Weight ($\underline{\mathbf{F}}_{\text{grav}}$)
- Normal force ($\underline{\mathbf{N}}$)
- Friction ($\underline{\mathbf{f}}_r$)

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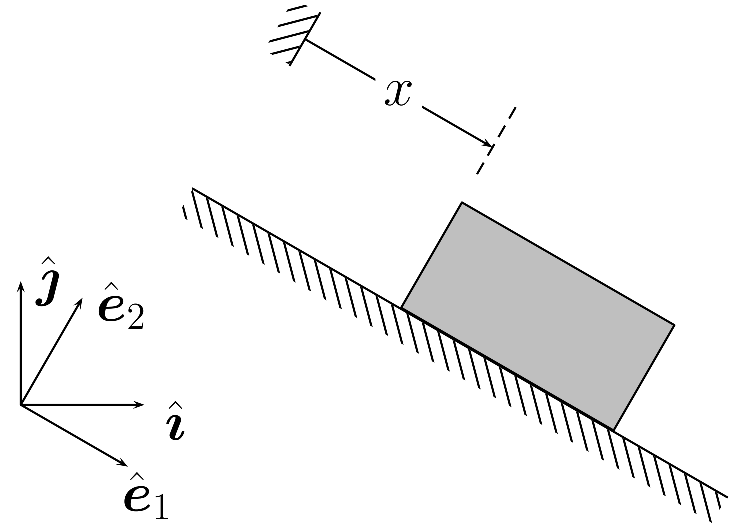
Coordinates and Directions/Kinematics

The motion of the block is only in the $\hat{e}_1$ direction, and we measure the displacement of the block as x. The directions $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}})$ and $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2)$ are related as

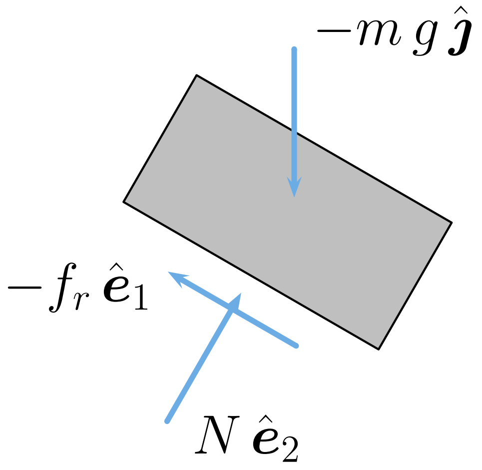
$$\hat{\boldsymbol{\imath}} = \mathrm{C}_\phi\ \hat{\boldsymbol{e}}_1 + \mathrm{S}_\phi\ \hat{\boldsymbol{e}}_2, \qquad \hat{\boldsymbol{\jmath}} = -\,\mathrm{S}_\phi\ \hat{\boldsymbol{e}}_1 + \mathrm{C}_\phi\ \hat{\boldsymbol{e}}_2.$$

With these, the acceleration of the block is

$$\underline{\mathbf{a}}_B = \ddot{x}\,\hat{\boldsymbol{e}}_1,$$



Free Body Diagram/Equations of Motion



Applying linear momentum balance to the block

$$\sum \underline{\mathbf{F}} = -m\,g\,\hat{\boldsymbol{\jmath}} - f_r\hat{\boldsymbol{e}}_1 + N\,\hat{\boldsymbol{e}}_2 = m\,\ddot{x}\,\hat{\boldsymbol{e}}_1 = m\,\underline{\mathbf{a}}_B,$$

$$\left(m\,g\ \mathrm{S}_\phi - f_r\right)\ \hat{\boldsymbol{e}}_1 + \left(-m\,g\ \mathrm{C}_\phi + N\right)\ \hat{\boldsymbol{e}}_2 = m\,\ddot{x}\,\hat{\boldsymbol{e}}_1,$$

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Taking components in the $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2)$ directions

$\hat{e}_1$ direction: $m\,g\,\mathrm{S}_\phi - f_r = m\,\ddot{x},$

$\hat{e}_2$ direction: $-m\,g\,\mathrm{C}_\phi + N = 0, \quad \longrightarrow \quad N = m\,g\,\mathrm{C}_\phi\,.$

If the block sticks, then $\ddot{x} \equiv 0$, so that the required friction force is

$$f_r = m\,g\,\mathrm{S}_\phi\,.$$

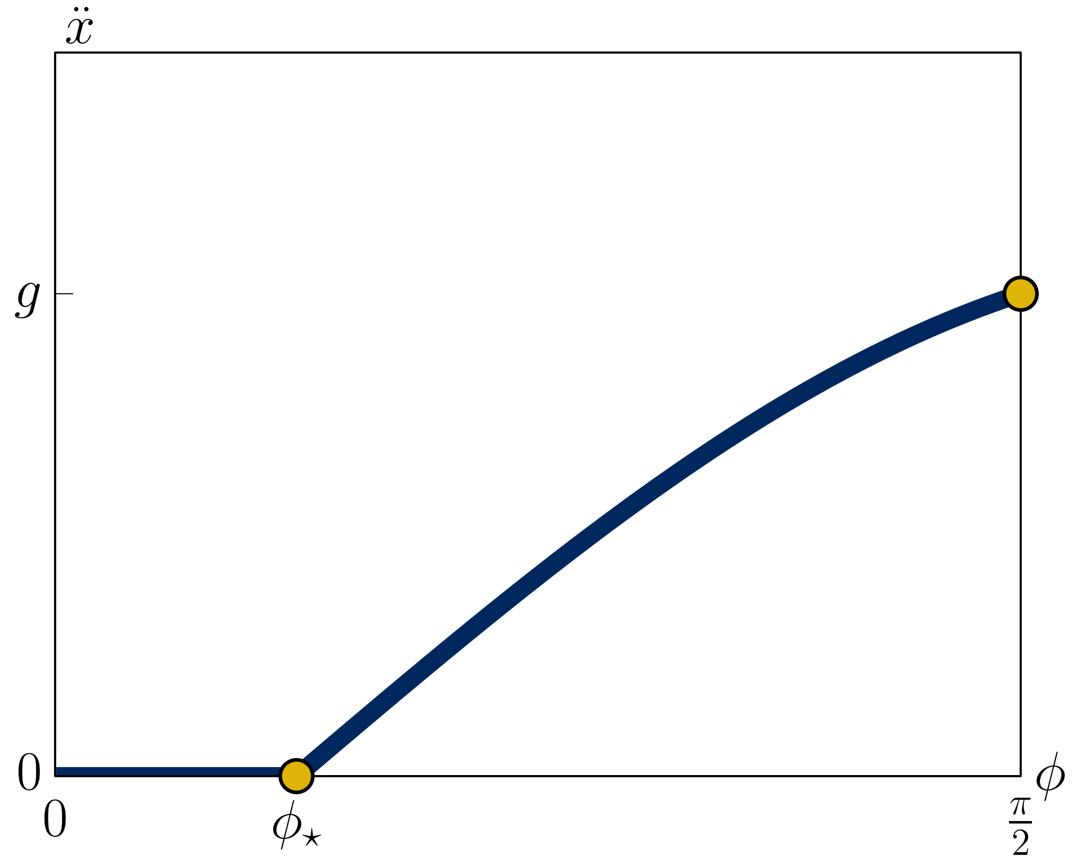
The magnitude of f_r must be less than $\mu\,N$, so that

$$|f_r| \le \mu\,N,$$
$$m\,g\,\mathrm{S}_\phi \le \mu\,m\,g\,\mathrm{C}_\phi, \quad \longrightarrow \quad \mathrm{T}_\phi \le \mu.$$

The critical angle of inclination is $\phi_\star = \mathrm{T}^{-1}_\mu$.

For $\phi > \phi_\star$ the block slides so that $f_r = \mu\,N$, and the acceleration can be solved as

$$\ddot{x} = g\,\mathrm{S}_\phi - \frac{\mu\,N}{m} = g\,\mathrm{S}_\phi - \mu\,g\,\mathrm{C}_\phi,$$
$$\longrightarrow \quad \underline{\mathbf{a}}_B = \ddot{x}\,\hat{\boldsymbol{e}}_1 = \left(g\,\left(\mathrm{S}_\phi - \mu\,\mathrm{C}_\phi\right)\right)\,\hat{\boldsymbol{e}}_1$$



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