Particle Dynamics

Engineering Mechanics: Dynamics

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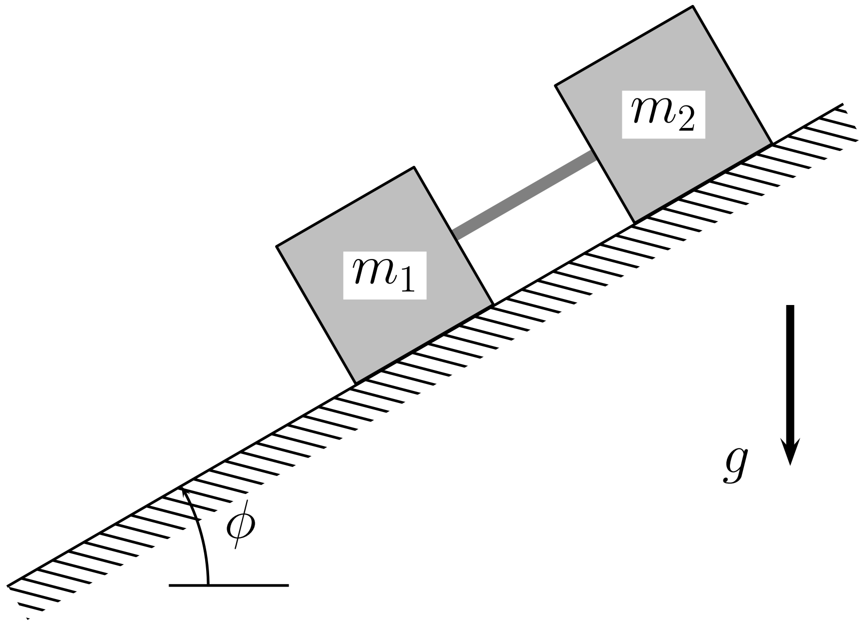
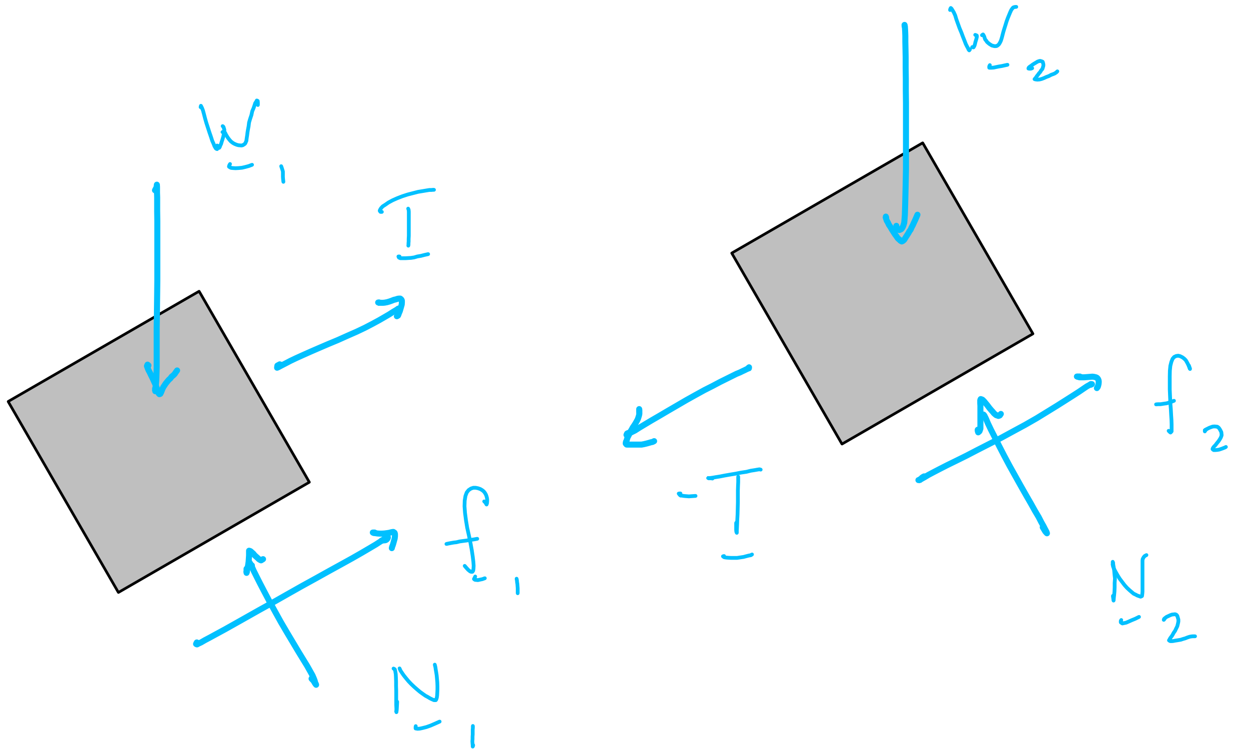
Two blocks of mass m_1 and m_2 are connected by a massless rod and slide down a rough surface inclined at an angle ϕ. The coefficient of friction between each block and the surface is μ_1 and μ_2. Find the acceleration of the pair and the tension in the rod. What is the minimum angle $\phi_\star$ so that the blocks slide?





FORCES

- WEIGHT ($\underline{W}$)
- NORMAL ($\underline{N}$)
- FRICTION ($\underline{f}$)
- TENSION ($\underline{T}$)

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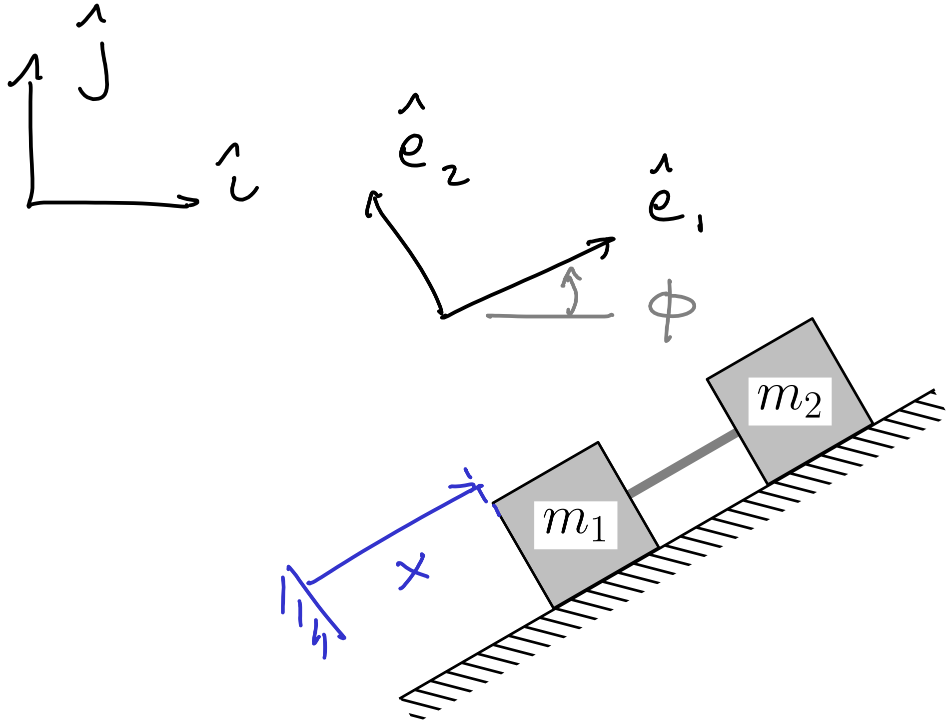
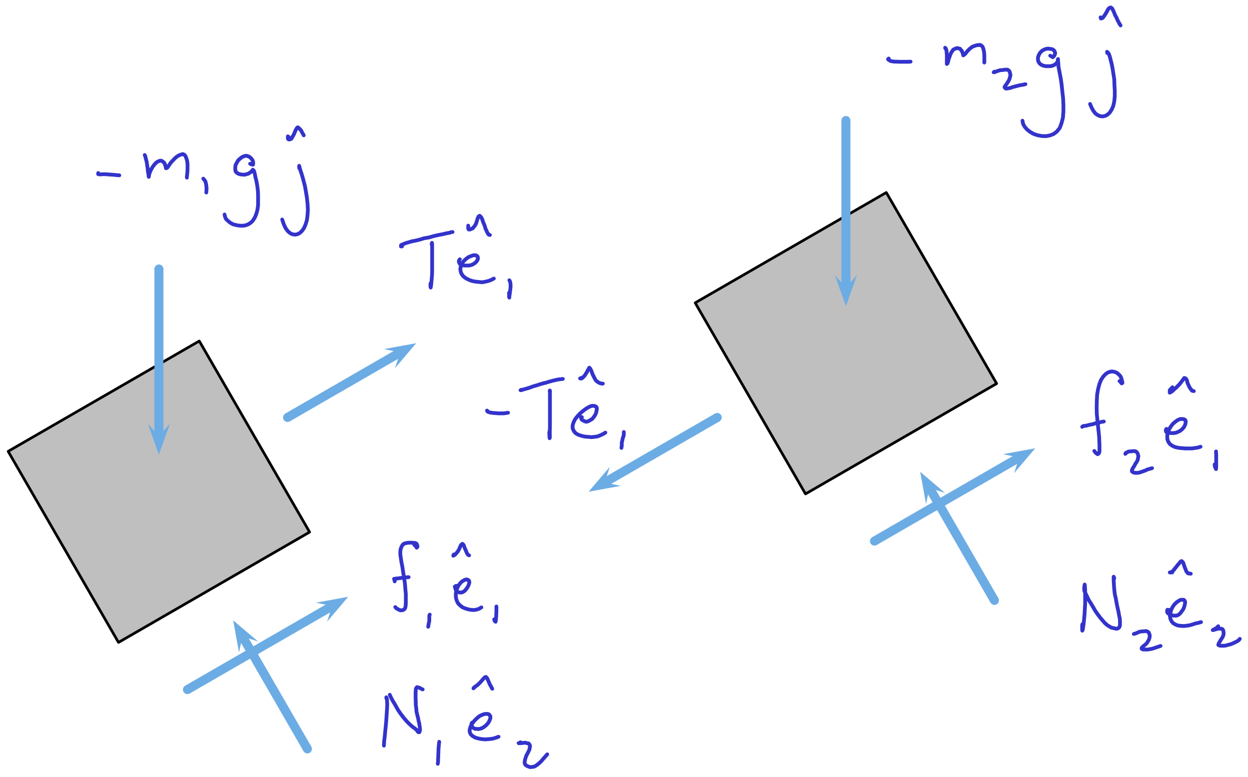
Coordinates and Directions

$$\hat{i} = C_\phi \hat{e}_1 - S_\phi \hat{e}_2 \qquad \hat{j} = S_\phi \hat{e}_1 + C_\phi \hat{e}_2$$



Kinematics/Free Body Diagram



ACCELERATION

$$a_1 = a_2 = \ddot{x}\,\hat{e}_1$$

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Equations of Motion

LINEAR MOMENTUM BALANCE

BLOCK 1: $-m_1 g\hat{j} + T\hat{e}_1 + f_1\hat{e}_1 + N_1\hat{e}_2 = \Sigma \underline{F}_1 = m_1\underline{a}_1 = m_1\ddot{x}\hat{e}_1$

BLOCK 2: $-m_2 g\hat{j} - T\hat{e}_1 + f_2\hat{e}_1 + N_2\hat{e}_2 = \Sigma \underline{F}_2 = m_2\underline{a}_2 = m_2\ddot{x}\hat{e}_1$

TAKE COMPONENTS IN $(\hat{e}_1, \hat{e}_2)$ DIRECTIONS

$\hat{j} = S_\phi \hat{e}_1 + C_\phi \hat{e}_2$

BLOCK 1

$\hat{e}_1$ DIR: $-m_1 g S_\phi + T + f_1 = m_1\ddot{x}$

$\hat{e}_2$ DIR: $-m_1 g C_\phi + N_1 = 0$

BLOCK 2

$\hat{e}_1$ DIR: $-m_2 g S_\phi - T + f_2 = m_2\ddot{x}$

$\hat{e}_2$ DIR: $-m_2 g C_\phi + N_2 = 0$

UNKNOWNS

$T, f_1, \ddot{x}, N_1, f_2, N_2$

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ELIMINATE T & SOLVE FOR (N_1, N_2)

$$N_1 = m_1 g C_\phi, \quad N_2 = m_2 g C_\phi, \qquad (m_1+m_2)\ddot{x} = -(m_1+m_2) g S_\phi + f_1 + f_2$$

&

$$T = m_1\ddot{x} - f + m_1 g S_\phi = \frac{m_1 f_2 - m_2 f_1}{(m_1+m_2)}$$

SLIPPING DOWN $(\dot{x} < 0)$

$$\left.\begin{aligned} f_1 &= \mu_1 N_1 = \mu_1 m_1 g C_\phi \\ f_2 &= \mu_2 N_2 = \mu_2 m_2 g C_\phi \end{aligned}\right\} \longrightarrow \quad T = \frac{(\mu_2 - \mu_1) m_1 m_2 g C_\phi}{m_1 + m_2}$$

& SO

$$\ddot{x} = -g S_\phi + \frac{(\mu_1 m_1 + \mu_2 m_2) g C_\phi}{(m_1+m_2)} \longrightarrow \quad \underline{a}_1 = \underline{a}_2 = \ddot{x}\,\hat{e}_1$$

<u>STICK</u> $\quad \ddot{x} = 0 \quad |f_i| \le \mu_i N_i$; @ $\phi = \phi_* \quad f_1 = \mu_1 N_1, \; f_2 = \mu_2 N_2$

$$0 = -(m_1+m_2) g S_{\phi_*} + (\mu_1 m_1 + \mu_2 m_2) g C_{\phi_*} \longrightarrow T_{\phi_*} = \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}$$

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