Cartesian Motion

Engineering Mechanics: Dynamics

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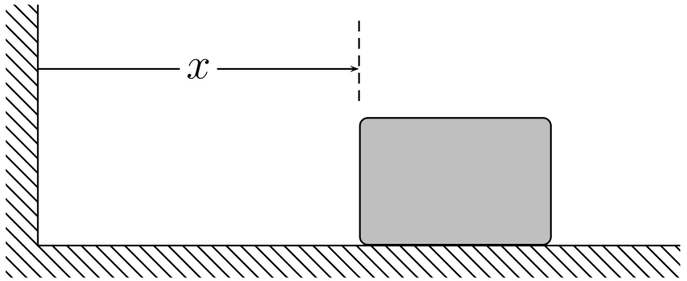
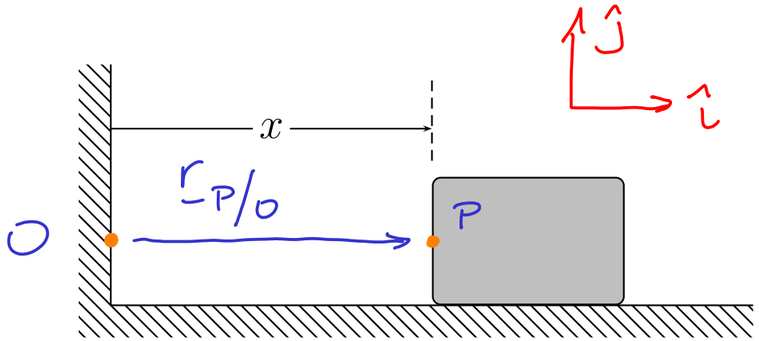
The acceleration of a block is given as

$$\ddot{x}(t) = \alpha\, e^{-\sigma\, t}.$$

If the block is initially displaced by a distance x_0 ($x(0) = x_0$) and released from rest ($\dot{x}(0) = 0$), find the velocity and position of the block as a function of time.





POSITION, VELOCITY, & ACCELERATION

$\underline{r}_{P/O} = x\,\hat{\imath}$, $\underline{v}_P = \dot{x}\,\hat{\imath}$, $\underline{a}_P = \ddot{x}\,\hat{\imath}$

$\ddot{x} = \alpha\, e^{-\sigma t}$ $\longrightarrow$

$$\int_0^t \left\{ \ddot{x}(\tau) = \alpha e^{-\sigma\tau} \right\} d\tau \longrightarrow \dot{x}(t) - \dot{x}(0) = -\frac{\alpha}{\sigma} e^{-\sigma\tau} \Big|_0^t$$

$$\dot{x}(t) = \dot{x}(0) + \frac{\alpha}{\sigma}\left(1 - e^{-\sigma t}\right)$$

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$$\int_0^t \left\{ \dot{x}(\tau) = \frac{\alpha}{\sigma}\left(1 - e^{-\sigma\tau}\right) \right\} dt \longrightarrow x(t) - x(0) = \frac{\alpha}{\sigma}\left(t + \frac{1}{\sigma}e^{-\sigma\tau}\right)\Big|_0^t$$

$$= \frac{\alpha}{\sigma}\left(t + \frac{1}{\sigma}\left(e^{-\sigma t} - 1\right)\right)$$

$$x(t) = x_0 + \frac{\alpha}{\sigma}\left(t + \frac{1}{\sigma}\left(e^{-\sigma t} - 1\right)\right)$$

$$\underline{v}_P = \dot{x}(t)\hat{\imath} = \left\{ \frac{\alpha}{\sigma}\left(1 - e^{-\sigma t}\right) \right\}\hat{\imath}$$

$$\underline{r}_{P/O} = x(t)\hat{\imath} = \left\{ x_0 + \frac{\alpha}{\sigma}\left(t + \frac{1}{\sigma}\left(e^{-\sigma t} - 1\right)\right) \right\}\hat{\imath}$$

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