Cartesian Motion

Engineering Mechanics: Dynamics

D. Dane Quinn, PhD

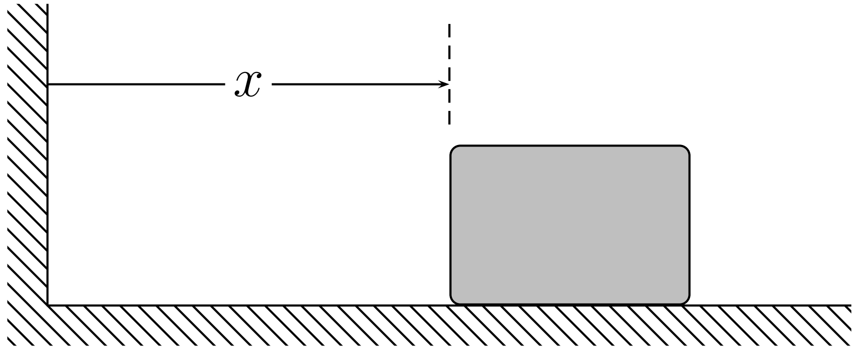
Department of Mechanical Engineering
The University of Akron
Akron OH 44325–3903 USA

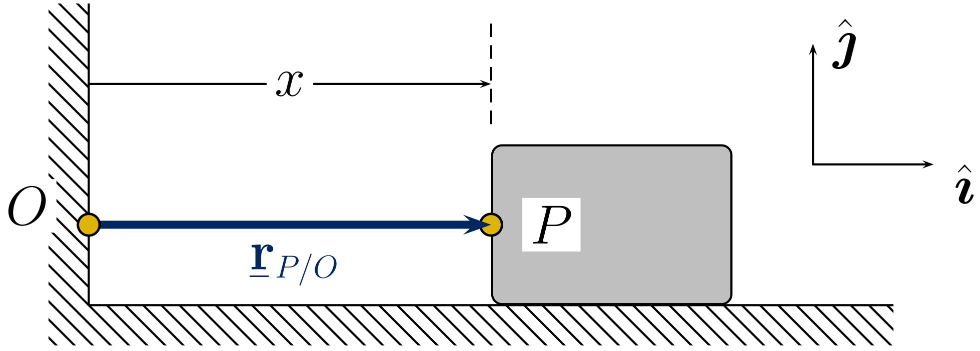
quinn@uakron.edu
www.uakron.edu

The acceleration of a block is given as

$$\ddot{x}(t) = \alpha\, e^{-\sigma\, t}.$$

If the block is initially displaced by a distance x_0 ($x(0) = x_0$) and released from rest ($\dot{x}(0) = 0$), find the velocity and position of the block as a function of time.





The position, velocity, and acceleration of the block can be expressed in terms of the coordinate x as

$$\underline{\mathbf{r}}_{P/O} = x\,\hat{\boldsymbol{\imath}}, \quad \underline{\mathbf{v}}_P = \dot{x}\,\hat{\boldsymbol{\imath}}, \quad \underline{\mathbf{a}}_P = \ddot{x}\,\hat{\boldsymbol{\imath}}.$$

Given $\ddot{x}$, we integrate with respect to time to find $\dot{x}$

$$\int_0^t \left\{ \ddot{x}(\tau) = \alpha\, e^{-\sigma\,\tau} \right\} d\tau \quad \longrightarrow \quad \begin{aligned} \dot{x}(t) - \dot{x}(0) &= -\frac{\alpha}{\sigma}\, e^{-\sigma\,\tau}\Big|_0^t, \\ &= \frac{\alpha}{\sigma}\left(1 - e^{-\sigma\, t}\right), \end{aligned}$$

so that

$$\dot{x}(t) = \frac{\alpha}{\sigma}\left(1 - e^{-\sigma\, t}\right).$$

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Integrating again provides the displacement x

$$\int_0^t \left\{ \dot{x}(\tau) = \frac{\alpha}{\sigma}\left(1 - e^{-\sigma\,\tau}\right) \right\} d\tau \quad \longrightarrow \quad \begin{aligned} x(t) - x(0) &= \frac{\alpha}{\sigma}\left(\tau + \frac{1}{\sigma}\,e^{-\sigma\,\tau}\right)\bigg|_0^t, \\ &= \frac{\alpha}{\sigma}\left(t + \frac{1}{\sigma}\left(e^{-\sigma\,t} - 1\right)\right), \end{aligned}$$

so that

$$x(t) = x_0 + \frac{\alpha}{\sigma}\left(t + \frac{1}{\sigma}\left(e^{-\sigma\,t} - 1\right)\right).$$

Finally,

$$\underline{\mathbf{v}}_P = \left\{\frac{\alpha}{\sigma}\left(1 - e^{-\sigma\,\tau}\right)\right\}\hat{\boldsymbol{\imath}}, \qquad \underline{\mathbf{r}}_{P/O} = \left\{x_0 + \frac{\alpha}{\sigma}\left(t + \frac{1}{\sigma}\left(e^{-\sigma\,t} - 1\right)\right)\right\}\hat{\boldsymbol{\imath}}.$$

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