# Cartesian Motion

## Engineering Mechanics: Dynamics

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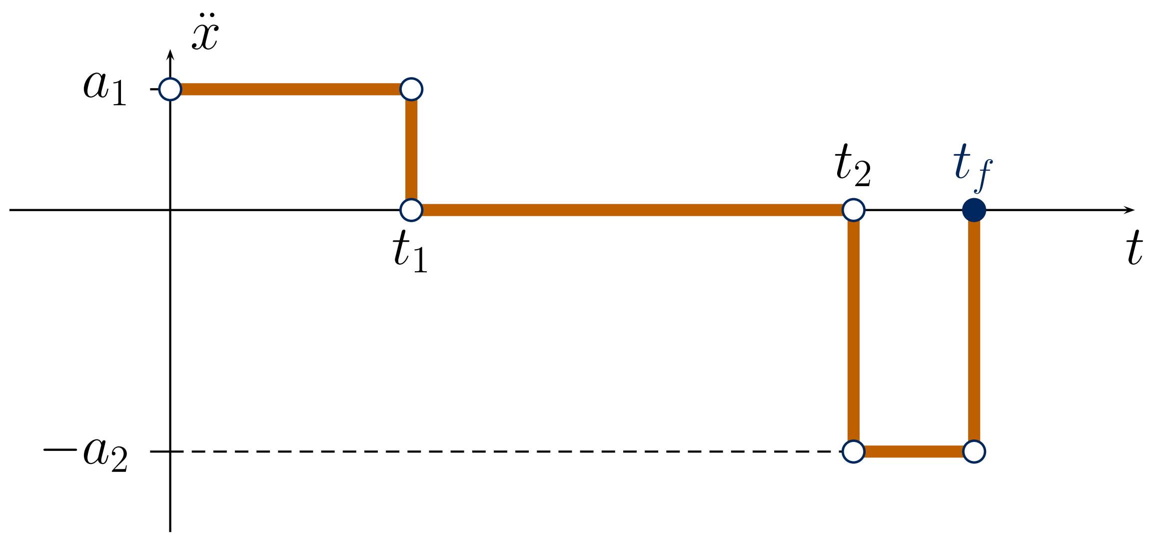
An automobile accelerates from rest for an initial time, holds a constant speed, and then decelerates and comes to rest as shown in the figure. Find the duration of the braking and the total distance traveled.




KNOWNS: $a_1, a_2, t_1, t_2$

$x(0) = 0$, $\dot{x}(0) = 0$, $\dot{x}(t_f) = 0$

UNKNOWNS: $t_f$, $x(t_f) = 0$

- INTEGRATE $\ddot{x} \longrightarrow \dot{x} \longrightarrow x$
- APPLY $\dot{x}(t_f) = 0 \longrightarrow t_f$
- FIND $x(t_f)$

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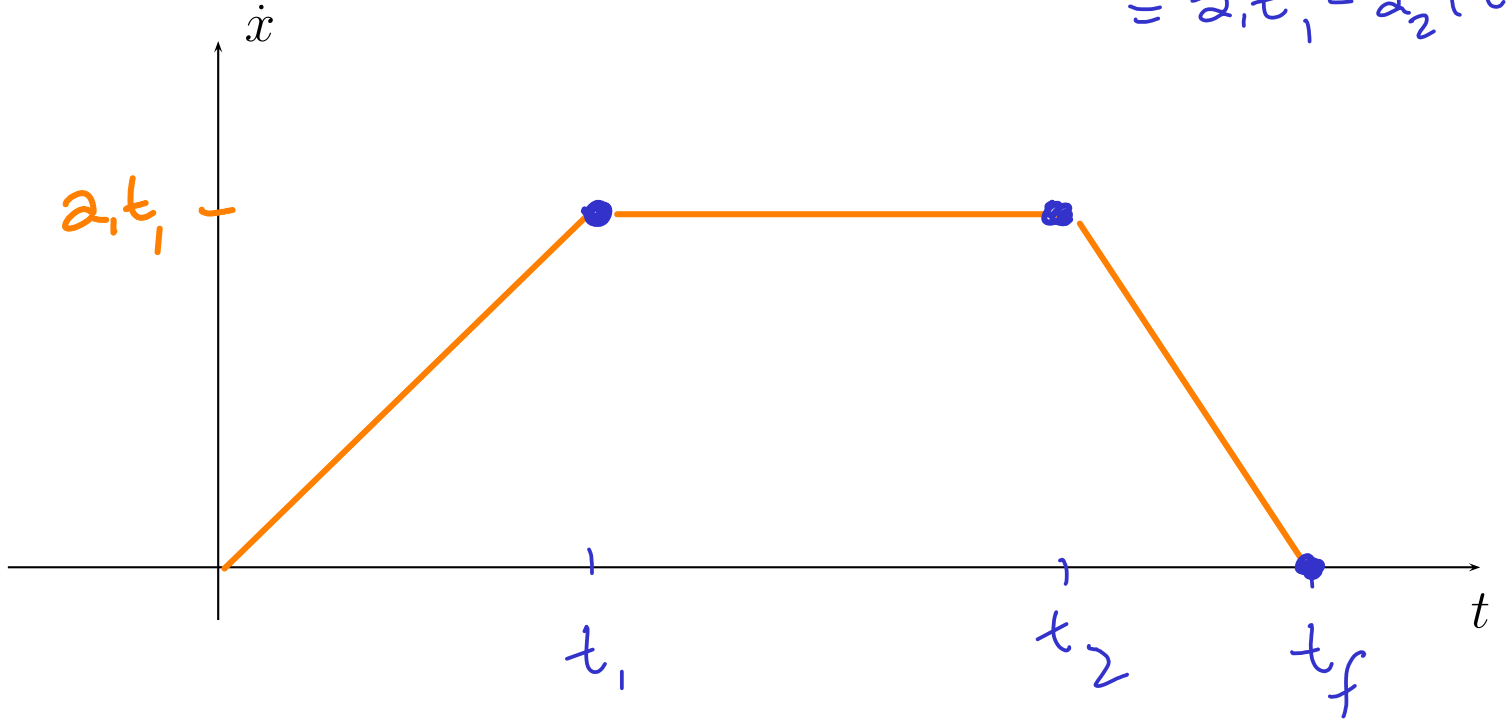
INTEGRATE $\ddot{x} \longrightarrow \int_0^t \ddot{x}(\tau)\,d\tau = \dot{x}(t) - \dot{x}(0)$

$$\dot{x}(t) - \dot{x}(0) = \begin{cases} \int_0^t a_1\,d\tau = a_1 t & 0 \le t < t_1 \\ \int_0^{t_1} a_1\,d\tau + \int_{t_1}^{t} 0\,d\tau = a_1 t_1 & t_1 \le t < t_2 \\ \int_0^{t_1} a_1\,d\tau + \int_{t_1}^{t_2} 0\,d\tau + \int_{t_2}^{t} (-a_1)\,d\tau = a_1 t_1 - a_2(t - t_2) & t_2 \le t < t_f \end{cases}$$




$$\dot{x}(t_f) = 0 = a_1 t_1 - a_2(t_f - t_2)$$

$$\longrightarrow t_f = t_2 + \frac{a_1 t_1}{a_2}$$

$$t_b = t_f - t_2 = \frac{a_1 t_1}{a_2}$$

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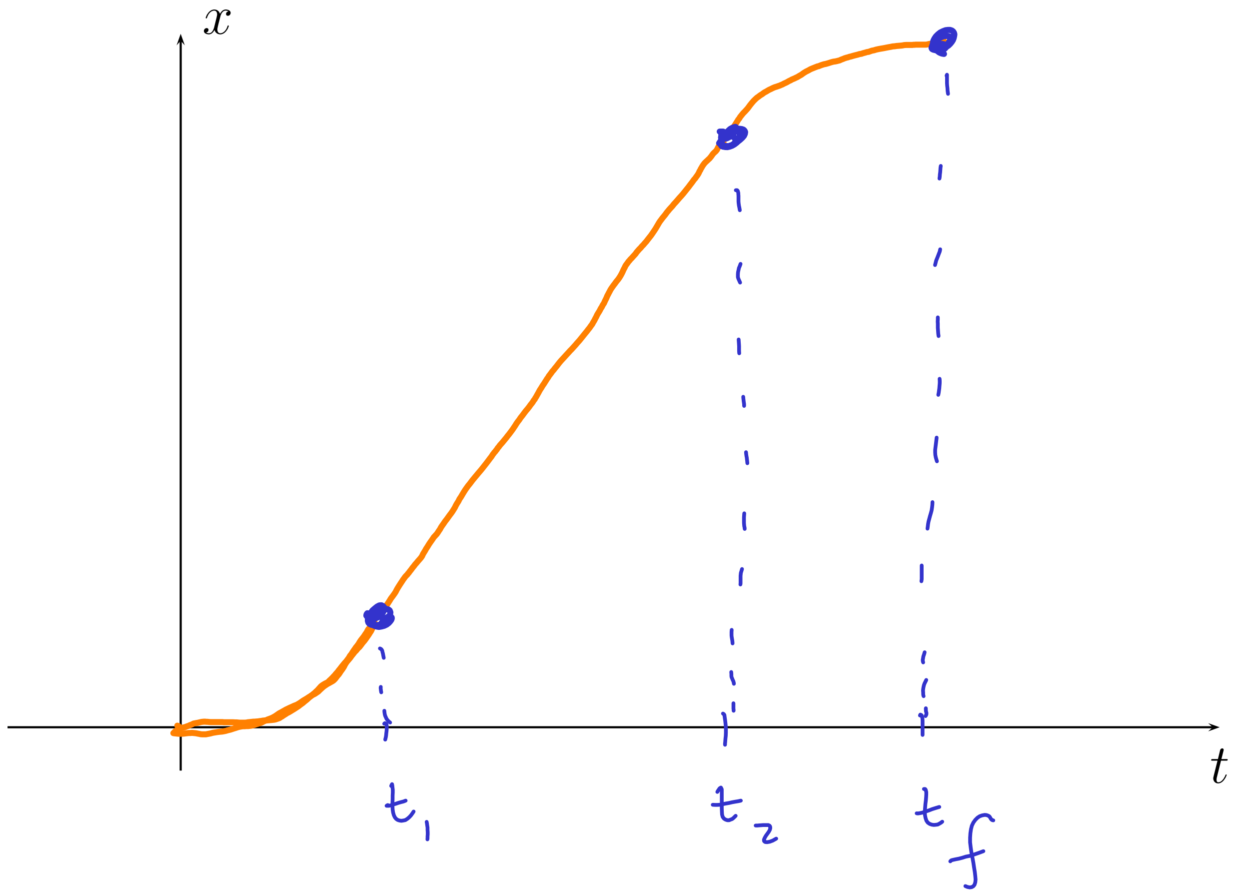
INTEGRATING AGAIN PROVIDES $x(t)$

$$x(t_f) - \cancel{x(0)}^{\,0} = \int_0^{t_f} \dot{x}(\tau)\,d\tau = \int_0^{t_1} a_1\tau\,d\tau + \int_{t_1}^{t_2} a_1 t_1\,d\tau + \int_{t_2}^{t_f} \left(a_1 t_1 - a_2(\tau - t_2)\right) d\tau$$

$$x(t_f) = \left[\frac{a_1}{2} t_1^2\right] + \left[a_1 t_1 (t_2 - t_1)\right] + \left[a_1 t_1 (t_f - t_2) - \frac{a_2}{2}(t_f - t_2)^2\right]$$

$$= \left[\frac{a_1}{2} t_1^2\right] + \left[a_1 t_1 (t_2 - t_1)\right] + \left[\frac{(a_1 t_1)^2}{2 a_2}\right]$$




$$t_f = t_2 + \frac{a_1 t_1}{a_2}$$

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