Cartesian Motion

Engineering Mechanics: Dynamics

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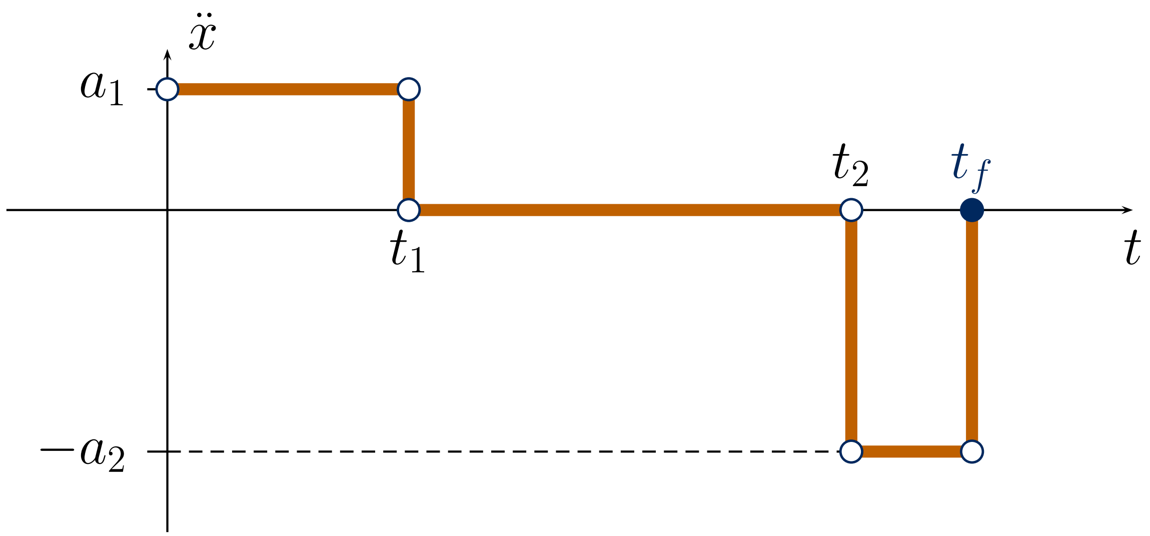
An automobile accelerates from rest for an initial time, holds a constant speed, and then decelerates and comes to rest as shown in the figure. Find the duration of the braking and the total distance traveled.



Here the values of a_1, a_2, t_1, and t_2 are known, and

$$\dot{x}(0) = 0, \quad x(0) = 0, \qquad \dot{x}(t_f) = 0,$$

while t_f and $x(t_f)$ are unknown.

- Integrate $\ddot{x}$ to find the velocity and position
- Apply the final velocity to solve for t_f (and the braking time)
- Determine the final displacement

The acceleration is defined piecewise, so the velocity and position must also be determined over these intervals.

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Integrate $\ddot{x} \longrightarrow \int_0^t \ddot{x}(\tau)\,d\tau = \dot{x}(t) - \dot{x}(0)$

$$\dot{x}(t) - \dot{x}(0) = \begin{cases} \displaystyle\int_0^t a_1\,d\tau = a_1\,t, & 0 \le t < t_1, \\ \displaystyle\int_0^{t_1} a_1\,d\tau + \int_{t_1}^{t} 0\,d\tau = a_1\,t_1, & t_1 \le t < t_2, \\ \displaystyle\int_0^{t_1} a_1\,d\tau + \int_{t_1}^{t_2} 0\,d\tau + \int_{t_2}^{t} -a_2\,d\tau = a_1\,t_1 - a_2\,(t - t_2), & t_2 \le t < t_f \end{cases}$$

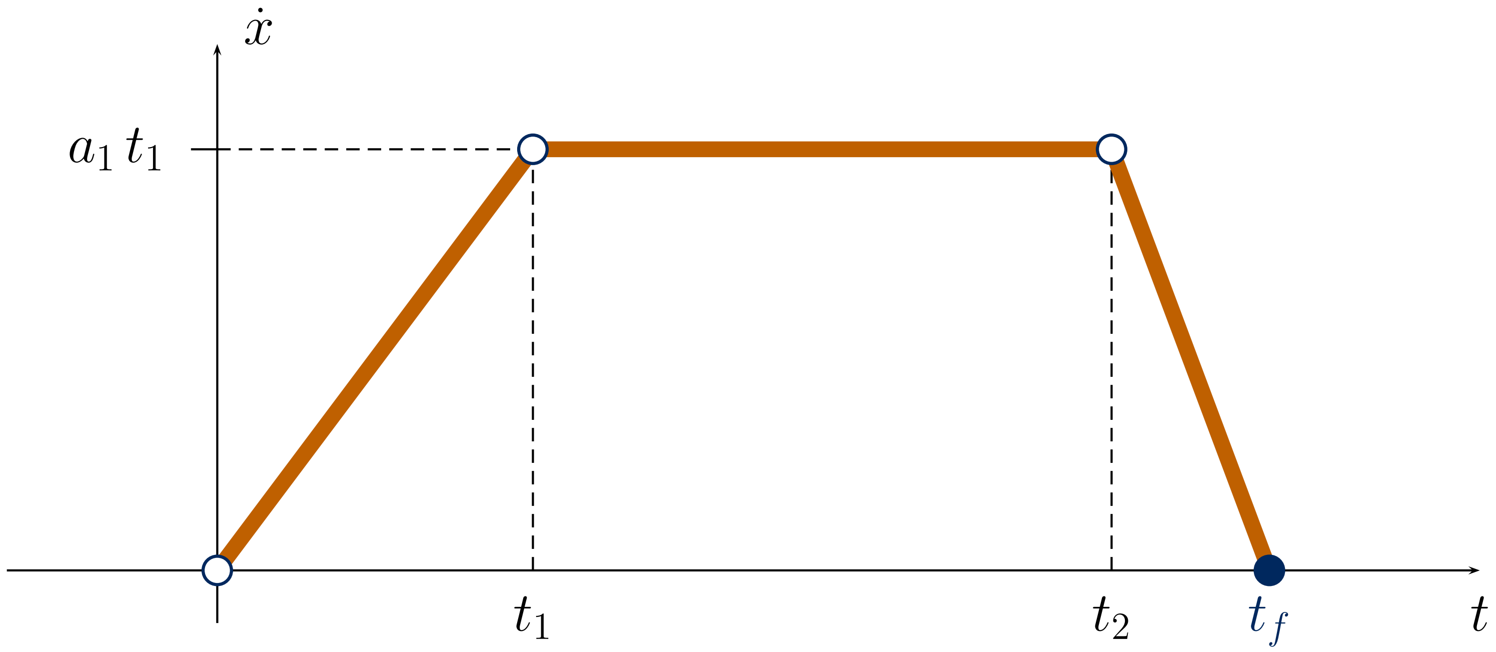
So

$$\dot{x}(t_f) = 0 = a_1\,t_1 - a_2\,(t - t_2),$$

$$\longrightarrow t_f = t_2 + \frac{a_1\,t_1}{a_2}.$$

The braking time is

$$t_b = t_f - t_2 = \frac{a_1\,t_1}{a_2}.$$



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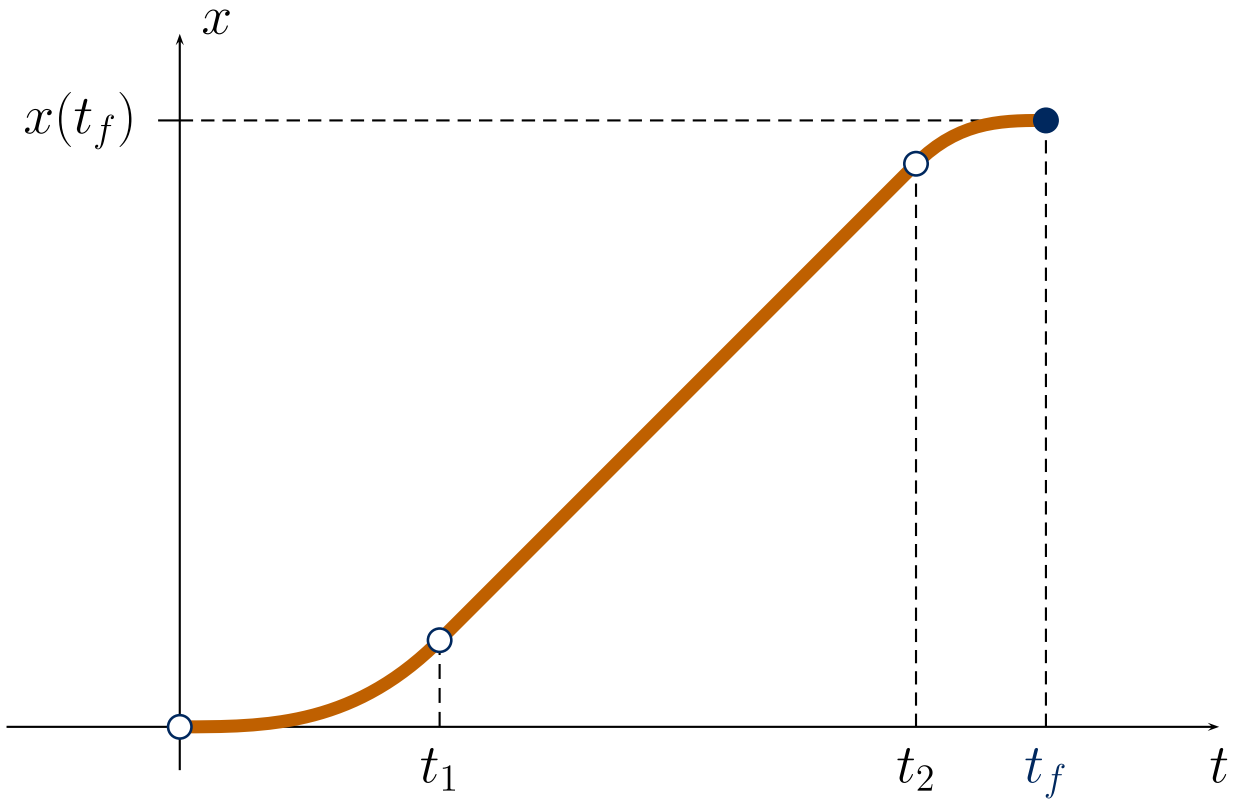
Integrating again provides the displacement $x(t_f)$

$$x(t) - x(0) = \int_0^{t_1} a_1 \, \tau \, d\tau + \int_{t_1}^{t_2} a_1 \, t_1 \, d\tau + \int_{t_2}^{t_f} \left(a_1 \, t_1 - a_2 \, (\tau - t_2)\right) \, d\tau,$$
$$= \left[\frac{a_1}{2} \, t_1^2\right] + \left[a_1 \, t_1 \, (t_2 - t_1)\right] + \left[a_1 \, t_1 \, (t_f - t_2) - \frac{a_2}{2} \, (t_f - t_2)^2\right],$$



so that

$$x(t_f) = \left[\frac{a_1 \, t_1^2}{2}\right] + \left[a_1 \, t_1 \, (t_2 - t_1)\right] + \left[\frac{(a_1 \, t_1)^2}{2 \, a_2}\right].$$

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