Polar Motion

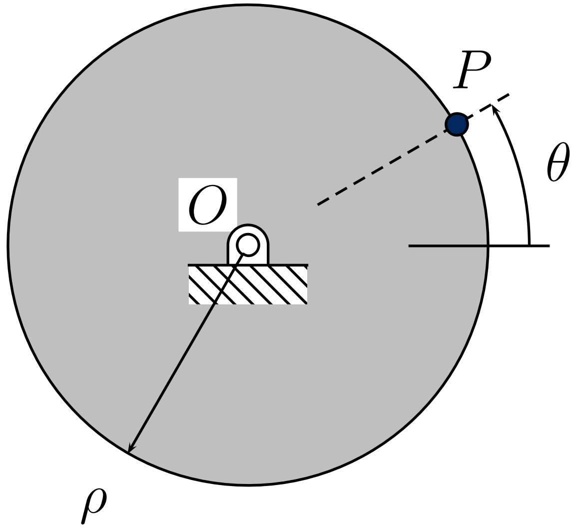
Engineering Mechanics: Dynamics

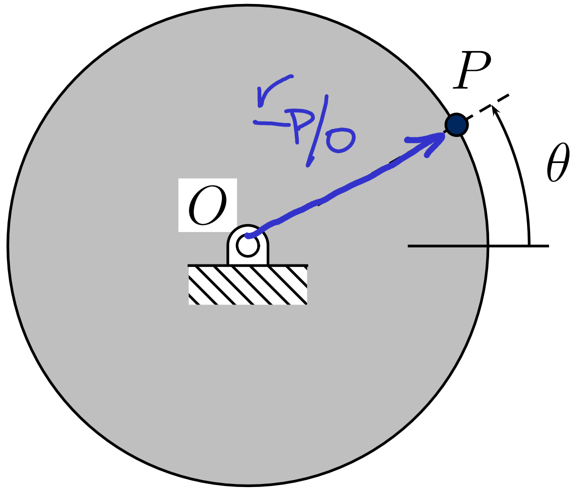
D. Dane Quinn, PhD

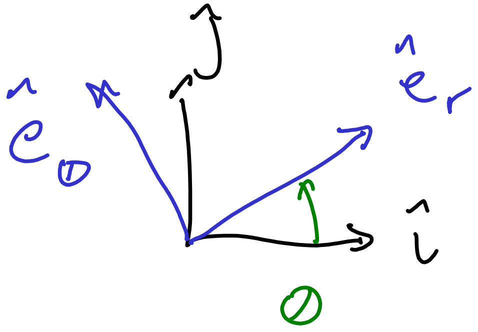
Department of Mechanical Engineering
The University of Akron
Akron OH 44325–3903 USA

quinn@uakron.edu
www.uakron.edu

A disk of radius ρ is pinned at its center and rotates with constant angular speed $\dot{\theta} = \omega$. Find the velocity and acceleration of point P, and the speed of that point.







RELATE $(\hat{i},\hat{j})$ TO $(\hat{e}_r,\hat{e}_\theta)$

$$\hat{e}_r = C_\theta \hat{i} + S_\theta \hat{j}$$

$$\hat{e}_\theta = -S_\theta \hat{i} + C_\theta \hat{j}$$

$$\hat{i} = C_\theta \hat{e}_r - S_\theta \hat{e}_\theta$$

$$\hat{j} = S_\theta \hat{e}_r + C_\theta \hat{e}_\theta$$

$$\int_0^t \left\{ \dot{\theta}(\tau) = \omega \right\} d\tau$$

$$\theta(t) = \omega t \qquad (\theta(0) = 0)$$

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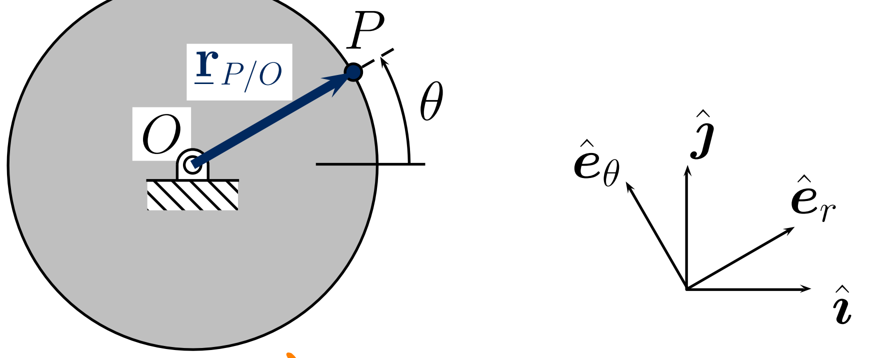
$$\underline{r}_{P/O} = r\,\hat{e}_r \longrightarrow \underline{v}_P = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$\underline{a}_P = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$



FOR THIS SYSTEM $r \equiv \rho$

$\dot{r} = 0$ $\quad$ $\ddot{r} = 0$ $\quad$ (CONSTANT RADIUS)

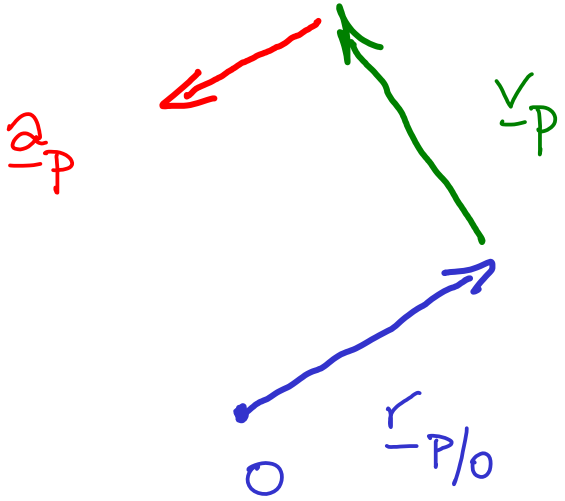
$\dot{\theta} = \omega$ $\quad$ $\ddot{\theta} = 0$ $\quad$ (CONSTANT ANGULAR SPEED)

SO THAT

$$\underline{v}_P = \rho\omega\,\hat{e}_\theta$$

$$\underline{a}_P = -\rho\omega^2\,\hat{e}_r$$

$\longrightarrow$ SPEED $\|\underline{v}_P\| = \rho\omega$



IN TERMS OF $(\hat{i}, \hat{j})$

$$\underline{v}_P = \rho\omega\left(-S_{\omega t}\,\hat{i} + C_{\omega t}\,\hat{j}\right)$$

$$\underline{a}_P = -\rho\omega^2\left(C_{\omega t}\,\hat{i} + S_{\omega t}\,\hat{j}\right)$$

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