Polar Motion

Engineering Mechanics: Dynamics

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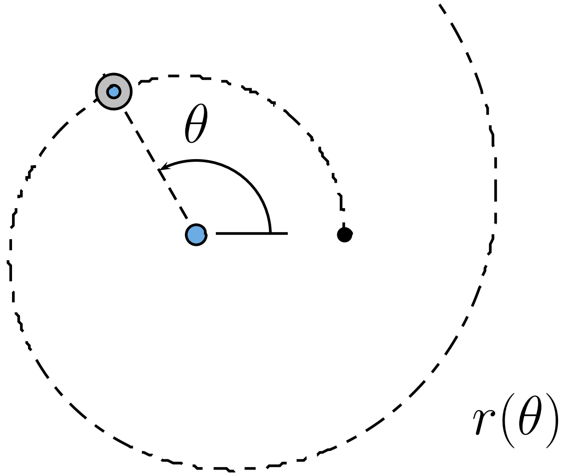
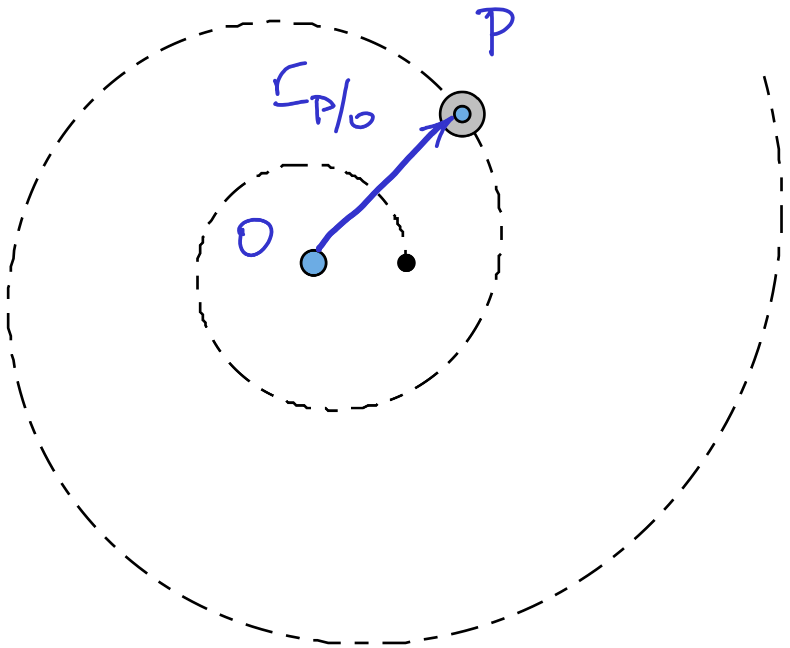
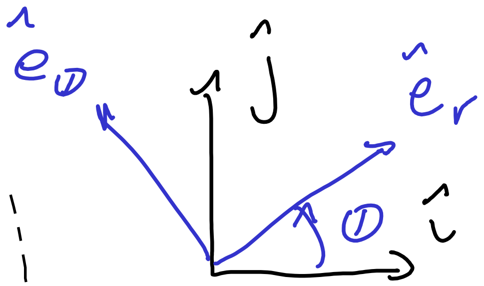
A particle moves on a helical path defined by

$$r(\theta) = k_0 + k_2\,\theta^2,$$

with $\dot{\theta} = \omega$. Find the position and velocity of the particle as a function of time if $\theta(0) = 0$.







POSITION

$$\underline{r}_{P/O} = r\,\hat{e}_r\,, \qquad \underline{v}_P = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$\hat{e}_r = C_\theta\,\hat{i} + S_\theta\,\hat{j} \qquad \hat{i} = C_\theta\,\hat{e}_r - S_\theta\,\hat{e}_\theta$$

$$\hat{e}_\theta = -S_\theta\,\hat{i} + C_\theta\,\hat{j} \qquad \hat{j} = S_\theta\,\hat{e}_r + C_\theta\,\hat{e}_\theta$$

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AS A FUNCTION OF TIME

$$\int_0^t \left\{ \dot{\theta}(\tau) = \omega \right\} d\tau \longrightarrow \theta(t) = \omega t \qquad (\theta(0) = 0)$$

CHAIN RULE

ALSO

$$r(t) = k_0 + k_2 (\theta(t))^2 \qquad \dot{r}(t) = \frac{d}{dt}\left(r(\theta(t))\right) = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = 2k_2\theta\dot{\theta}$$

$$= k_0 + k_2(\omega t)^2 \qquad\qquad = 2k_2\omega^2 t$$

THEREFORE

$$r_{P/O}(t) = r(t)\hat{e}_r(t) = \left(k_0 + k_2\omega^2 t^2\right)\hat{e}_r(t)$$

$$= \left(\left(k_0 + k_2\omega^2 t^2\right) C_{\omega t}\right)\hat{i} + \left(\left(k_0 + k_2\omega^2 t^2\right) S_{\omega t}\right)\hat{j}$$

$$v_P(t) = \dot{r}(t)\hat{e}_r(t) + r(t)\dot{\theta}(t)\hat{e}_\theta(t) = \left(2k_2\omega^2 t\right)\hat{e}_r(t) + \left(k_0 + k_2\omega^2 t^2\right)\omega\,\hat{e}_\theta(t)$$

$$= \left(\left(2k_2\omega^2 t\right) C_{\omega t} - \left(k_0 + k_2\omega^2 t^2\right)\omega S_{\omega t}\right)\hat{i}$$

$$+ \left(\left(2k_2\omega^2 t\right) S_{\omega t} + \left(k_0 + k_2\omega^2 t^2\right)\omega C_{\omega t}\right)\hat{j}$$

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