Motion

Engineering Mechanics: Dynamics

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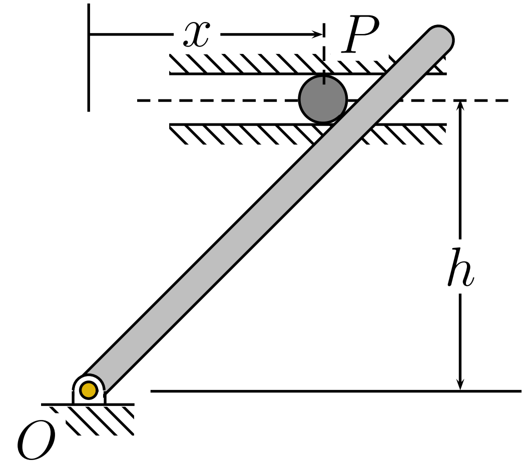
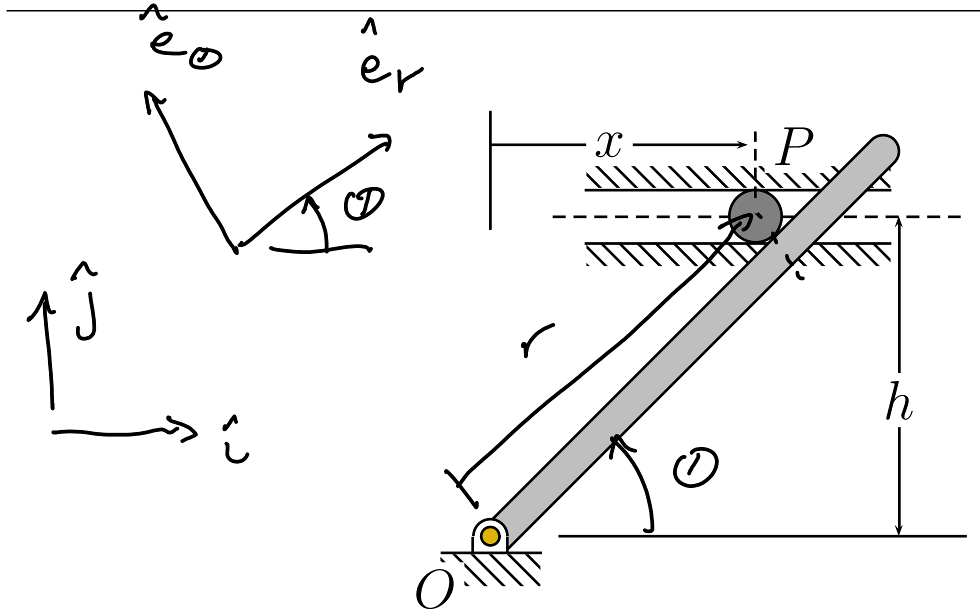
A ball moves along a horizontal slot, pushed by a rotating bar. Find the angular velocity and angular acceleration of the bar as a function of x so that the ball moves to the left with constant speed v.





$$\hat{e}_r = C_\theta \hat{i} + S_\theta \hat{j}$$
$$\hat{e}_\theta = -S_\theta \hat{i} + C_\theta \hat{j}$$

$$\hat{i} = C_\theta \hat{e}_r - S_\theta \hat{e}_\theta$$
$$\hat{j} = S_\theta \hat{e}_r + C_\theta \hat{j}$$

$$\underline{\omega}_B = \dot{\theta}\hat{k}, \quad \underline{\alpha}_B = \ddot{\theta}\hat{k}$$

CARTESIAN

$$\underline{r}_{P/O} = x\hat{i} + h\hat{j}, \quad \underline{v}_P = \dot{x}\hat{i} = -v\hat{i}, \quad \underline{a}_P = \ddot{x}\hat{i} = \underline{0}$$

POLAR

$$\underline{r}_{P/O} = r\hat{e}_r, \quad \underline{v}_P = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta, \quad \underline{a}_P = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

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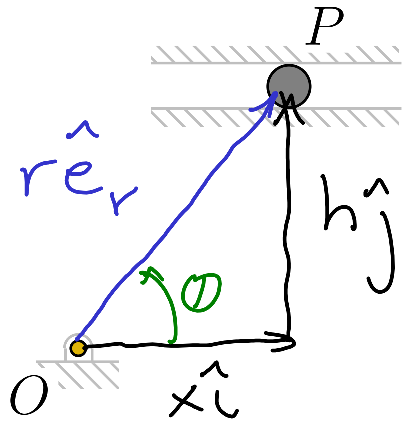
VELOCITY

$$\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = \underline{v}_P = \dot{x}\hat{\imath}$$
$$= -v\hat{\imath} = (-vC_\theta)\hat{e}_r + (vS_\theta)\hat{e}_\theta \longrightarrow \dot{r} = -vC_\theta, \quad \dot{\theta} = \frac{vS_\theta}{r}$$

POSITION



$$r\hat{e}_r = \underline{r}_{P/O} = x\hat{\imath} + h\hat{j}$$
$$(rC_\theta)\hat{\imath} + (rS_\theta)\hat{j} = x\hat{\imath} + h\hat{j} \longrightarrow r = \sqrt{x^2+h^2}, \quad T_\theta = \frac{h}{x}$$
$$S_\theta = \frac{h}{r} = \frac{h}{\sqrt{x^2+h^2}}; \quad C_\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+h^2}}$$

IN TERMS OF x

$$\dot{r} = -v\frac{x}{r} = \frac{-vx}{\sqrt{x^2+h^2}}, \quad \dot{\theta} = v\frac{(h/r)}{r} = \frac{vh}{x^2+h^2} \longrightarrow \underline{\omega}_B = \dot{\theta}\hat{k} = \left(\frac{vh}{x^2+h^2}\right)\hat{k}$$

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ACCELERATION

$$(\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta = \underline{a}_p = \ddot{x}\,\hat{i}$$

$$= \underline{0} \longrightarrow \quad \ddot{r} - r\dot{\theta}^2 = 0, \quad r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$$

SOLVE

$$\ddot{r} = r\dot{\theta}^2 = r\left(\frac{vh}{r^2}\right)^2 = \frac{v^2h^2}{r^3}$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -\frac{2\left(-\frac{vx}{r}\right)\left(\frac{vh}{r^2}\right)}{r} = \frac{2v^2xh}{r^4}$$

IN TERMS OF x

$$\ddot{r} = \frac{v^2h^2}{(x^2+h^2)^{3/2}}; \quad \ddot{\theta} = \frac{2v^2xh}{(x^2+h^2)^2} \longrightarrow \underline{\alpha}_B = \ddot{\theta}\hat{k} = \left(\frac{2v^2xh}{(x^2+h^2)^2}\right)\hat{k}$$

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