Planar Dynamics

Engineering Mechanics: Dynamics

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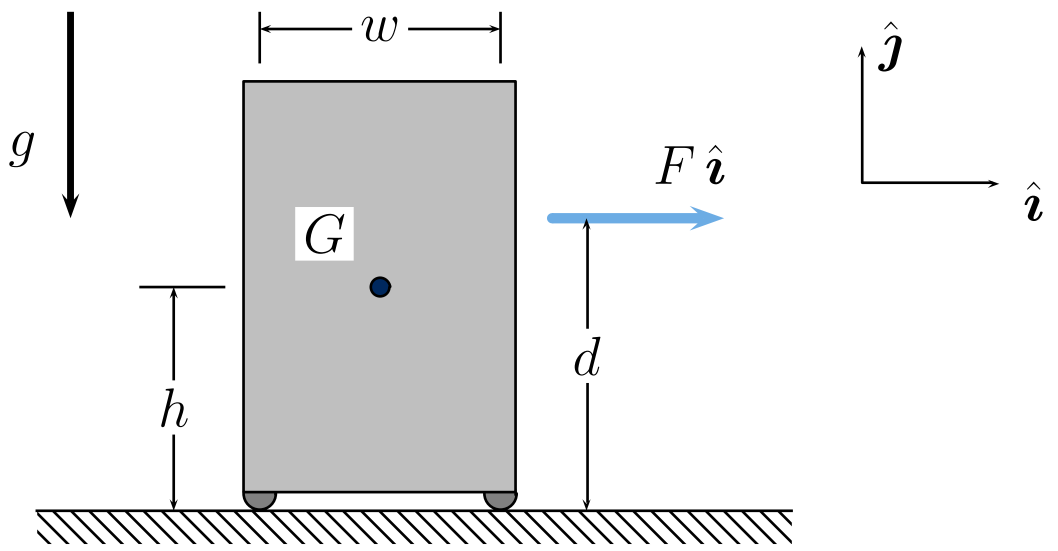
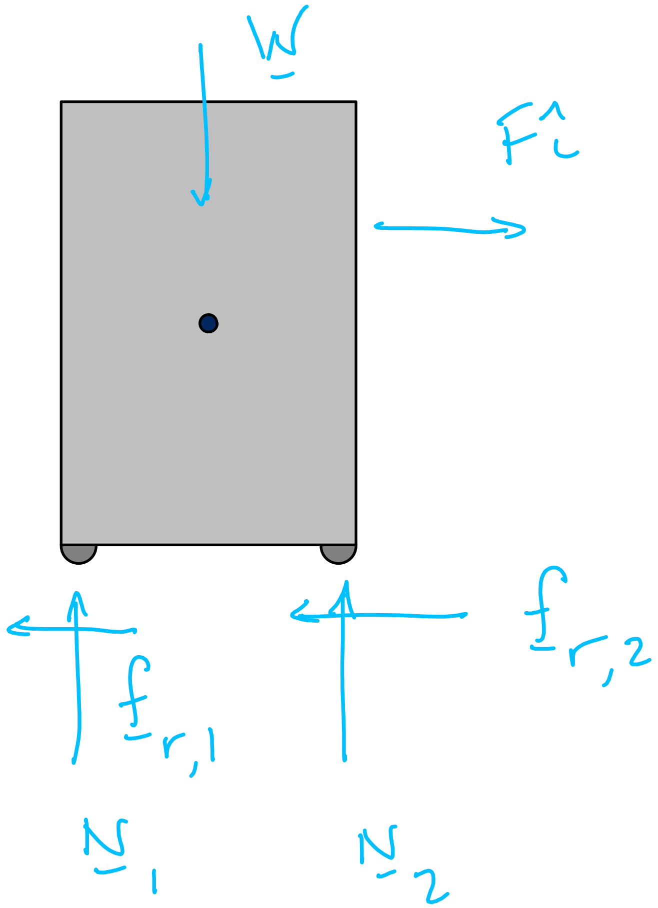
A block of mass m is in contact with a rough surface (coefficient of friction μ), and is pulled by a horizontal force of magnitude $F > 0$ applied at height d. The mass center of the block is located at height h and is centered between the supports, located a distance w apart. Find the maximum value of F so that the block does not tip when released from rest.





FORCES ARISE FROM

- NORMAL LOADS ($\underline{N}_1, \underline{N}_2$)
- FRICTION ($\underline{f}_{r,1}, \underline{f}_{r,2}$)
- WEIGHT ($\underline{W}$)
- APPLIED FORCE ($F\hat{i}$)

BLOCK RESTS ON SURFACE

↓

NORMAL FORCES ARE IN $+\hat{j}$

ASSUME THAT THE BLOCK DOES NOT ROTATE & FIND F_{MAX} SO THAT EITHER $\underline{N}_1$ OR $\underline{N}_2$ IS ADHESIVE

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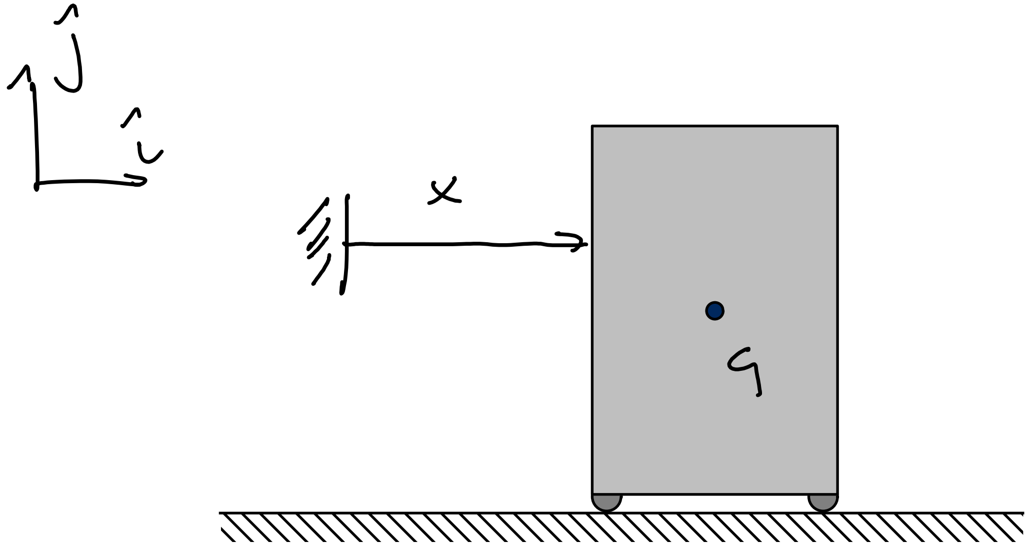
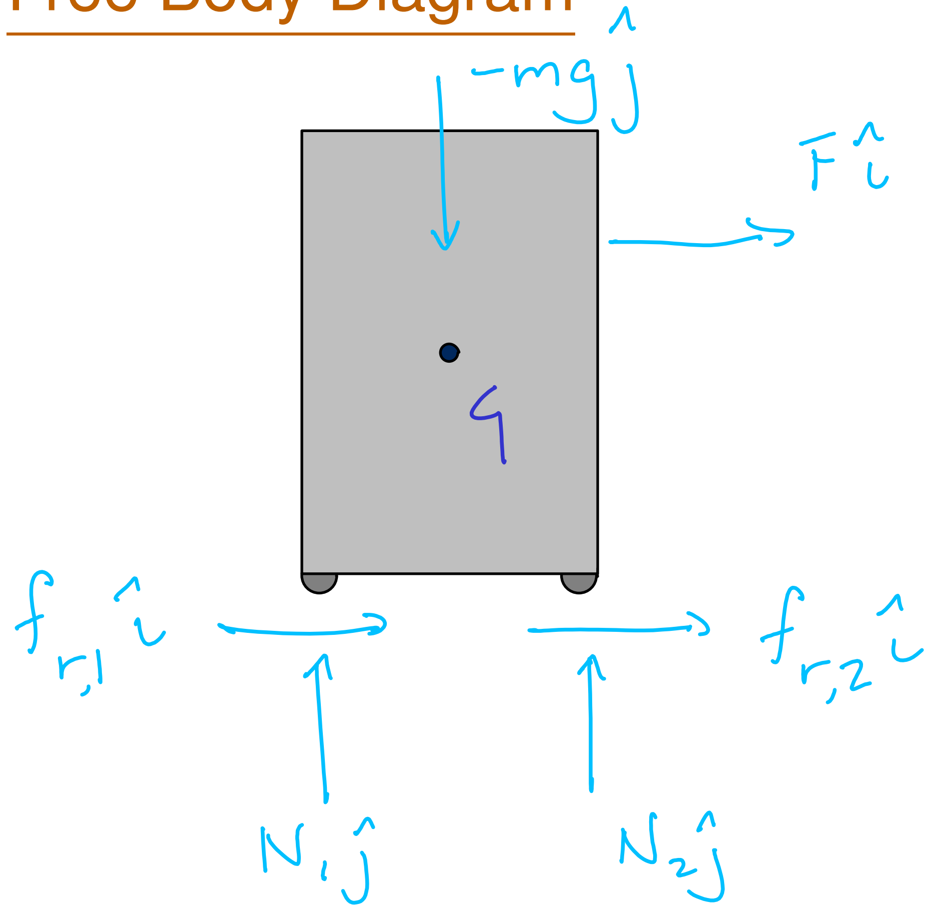
Coordinates and Directions/Kinematics

MEASURE THE DISPLACEMENT OF THE BLOCK AS x

$\underline{\alpha}_B = \underline{0} \qquad \underline{a}_G = \ddot{x}\,\hat{\imath}$



Free Body Diagram



DEFINE FRICTION

$$f_r = -\mu N \cdot \text{SIGN}(\dot{x}), \qquad |\dot{x}| > 0$$

$$= f_{\text{STICK}} \qquad \dot{x} = 0$$

FORCE REQUIRED TO MAINTAIN EQUILIBRIUM

$$|f_r| \le \mu N$$

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Equations of Motion

$\sum \underline{F} = m \underline{a}_G$

$$(F + f_{r,1} + f_{r,2})\,\hat{i} + (-mg + N_1 + N_2)\,\hat{j} = m\ddot{x}\,\hat{i} + 0\,\hat{j}$$

$\sum \underline{M}_G = I_G \underline{\alpha}_\beta$

$$\left(-F(d-h) + f_{r,1}h + f_{r,2}h - N_1\frac{w}{2} + N_2\frac{w}{2}\right)\hat{k} = \underline{0}$$

TAKING COMPONENTS

$$F + f_{r,1} + f_{r,2} = m\ddot{x} \qquad -mg + N_1 + N_2 = 0$$

$$-F(d-h) + f_{r,1}h + f_{r,2}h - N_1\frac{w}{2} + N_2\frac{w}{2} = 0$$

WE MUST CONSIDER TWO STATES OF MOTION

STICK – $\ddot{x} = 0$ $\quad |f_{r,i}| \leq \mu N_i$

SLIP – $|\dot{x}| > 0$ $\quad |f_{r,i}| = \mu N_i$

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STICK

$\dot{x} \equiv 0 \longrightarrow \ddot{x} = 0$

$$F + f_{r,1} + f_{r,2} = 0 \qquad N_1 + N_2 - mg = 0$$

$$-F(d-h) + f_{r,1}h + f_{r,2}h - N_1\frac{w}{2} + N_2\frac{w}{2} = 0$$

$$f_{r,1} + f_{r,2} = -F \qquad N_1 + N_2 = mg$$

$$-F\left((d-\cancel{h}) + \cancel{h}\right) - (N_1 - N_2)\frac{w}{2} = 0$$

$$N_1 = \frac{mg}{2} - \frac{Fd}{w} \qquad N_2 = \frac{mg}{2} + \frac{Fd}{w}$$

$$|f_r| < \mu N$$

$$F = |f_{r,1} + f_{r,2}| \leq \mu(N_1 + N_2) = \mu mg$$

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BOTH N_1 & N_2 MUST BE POSITIVE @ $F = F_*$, $N_1 = 0$

$$N_1 = \frac{mg}{2} - \frac{F_* d}{w} = 0 \longrightarrow F_* = \frac{mgw}{2d} \qquad F \leq \mu mg$$

SLIP

$F > \mu mg \longrightarrow \ddot{x} > 0$ WITH $f_{r,1} = -\mu N_1$ $\quad f_{r,2} = -\mu N_2$

$$F - \mu(N_1 + N_2) = m\ddot{x} \qquad N_1 + N_2 = mg$$

$$-F(d-h) - \mu(N_1 + N_2)h - (N_1 - N_2)\frac{w}{2} = 0$$

& SO

$$\ddot{x} = F - \mu mg$$

$$N_1 = \frac{mg}{2} - \frac{F(d-h) + \mu mgh}{w} \quad ; \quad N_2 = \frac{mg}{2} + \frac{F(d-h) + \mu mgh}{w}$$

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BOTH N_1 & N_2 MUST BE POSITIVE

$(d-h) > 0$

$F \leq \mu mg \qquad \left(\frac{F}{mg} < \mu\right)$

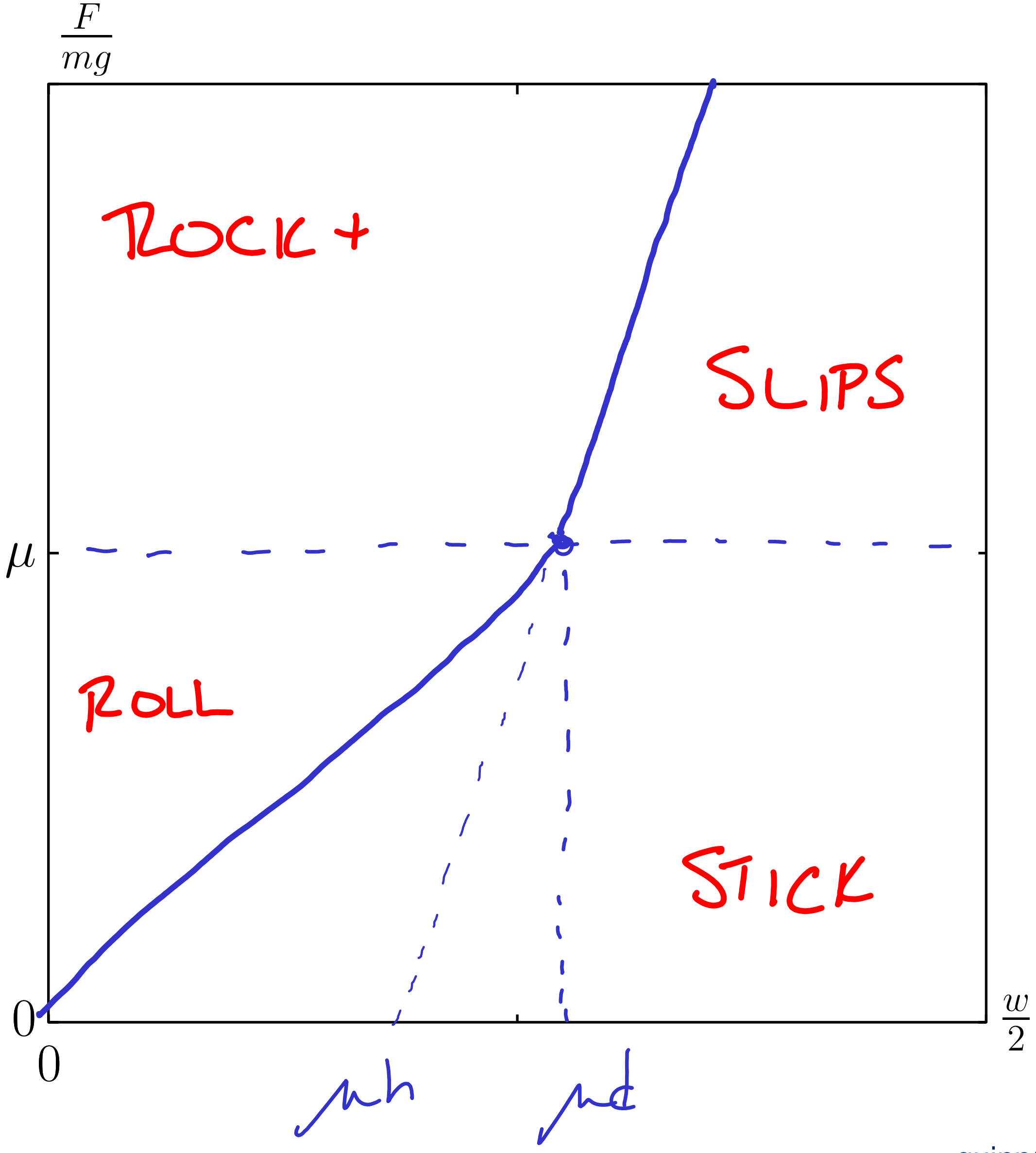
$\frac{F_*}{mg} = \frac{w/2}{d} \qquad \frac{F}{mg} < \frac{F_*}{mg} = \frac{w/2}{d}$ STICK

$F > \mu mg \qquad \left(\frac{F}{mg} > \mu\right) \qquad N_2 > 0$

$N_1 = 0 \quad @ \quad F = F_*$

$$N_1 = \frac{mg}{2} - \frac{F_*(d-h) + \mu mgh}{w} = 0$$

$$\frac{F_*}{mg} = \frac{1}{(d-h)}\left(\frac{w}{2} - \mu h\right)$$



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$d-h<0$

$$N_1 = \frac{mg}{2} + \frac{F(h-d) - \mu mgh}{w} \qquad N_2 = \frac{mg}{2} - \frac{F(h-d) - \mu mgh}{w}$$

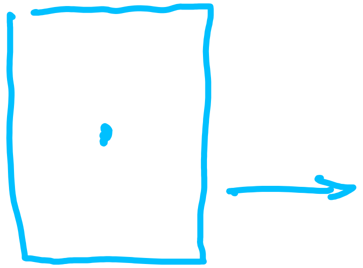
$F \le \mu mg$ BLOCK TIPS AT $F = F_*$

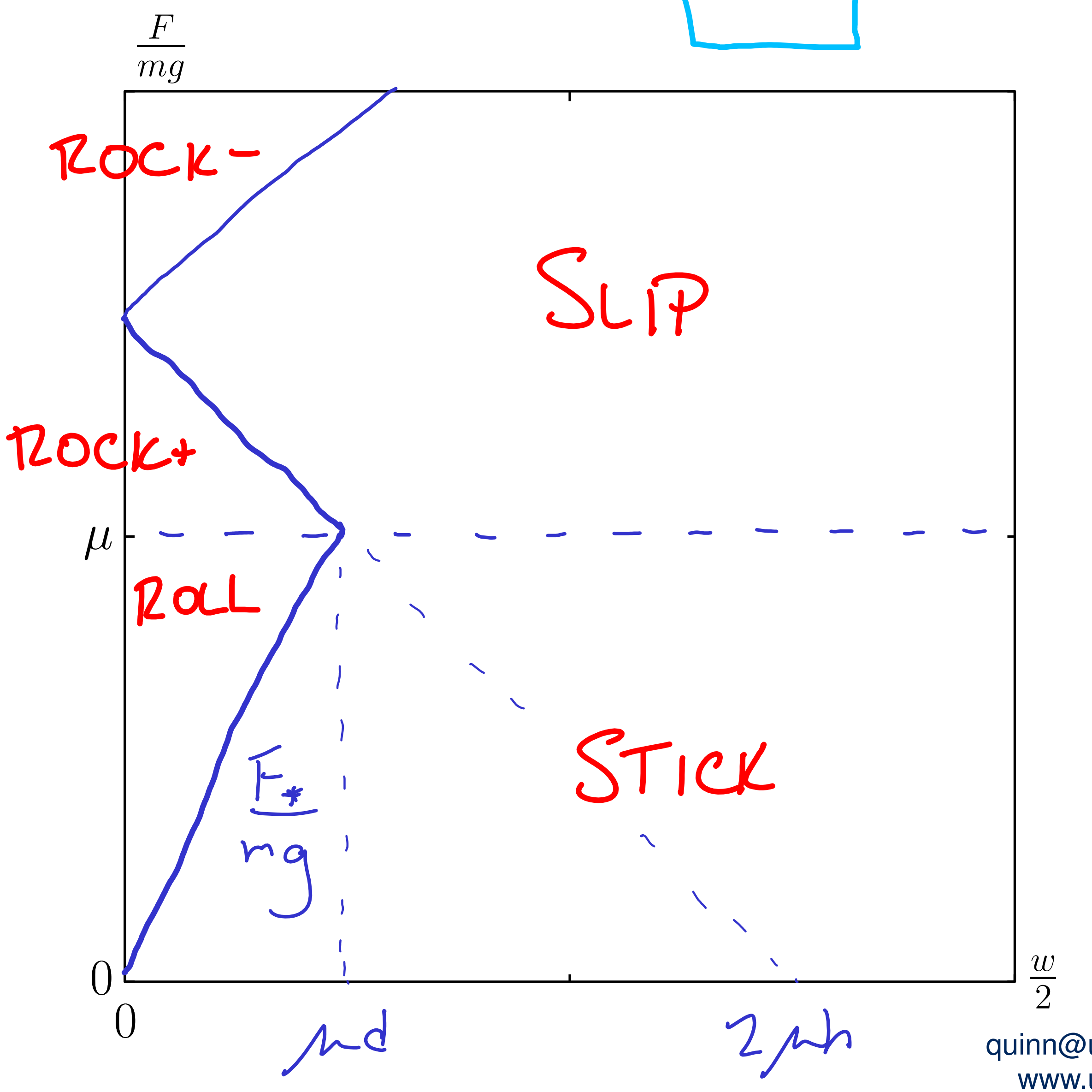
$$\frac{F_*}{mg} = \frac{w/2}{d}$$

$F > \mu mg$ $N_1 = 0$ OR $N_2 = 0$ @ $F = F_*$

$$N_1 = 0 \longrightarrow \frac{F_*}{mg} = \frac{1}{h-d}\left(\mu h - \frac{w}{2}\right)$$

$$N_2 = 0 \longrightarrow \frac{F_*}{mg} = \frac{1}{h-d}\left(\mu h + \frac{w}{2}\right)$$





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