Rigid Body Dynamics

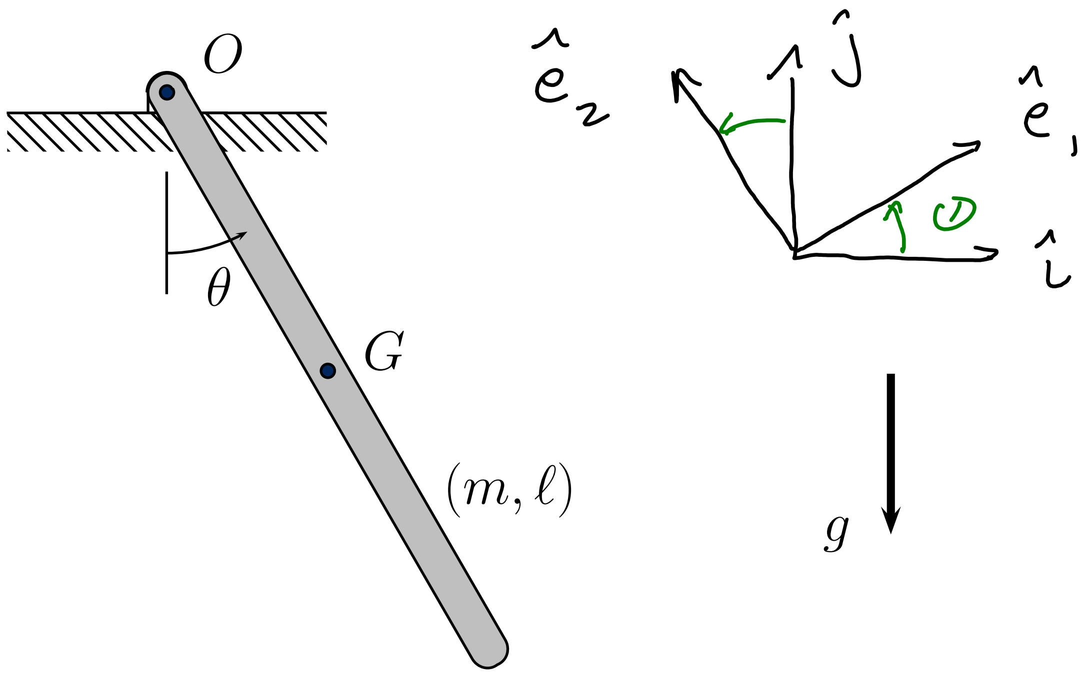
Engineering Mechanics: Dynamics

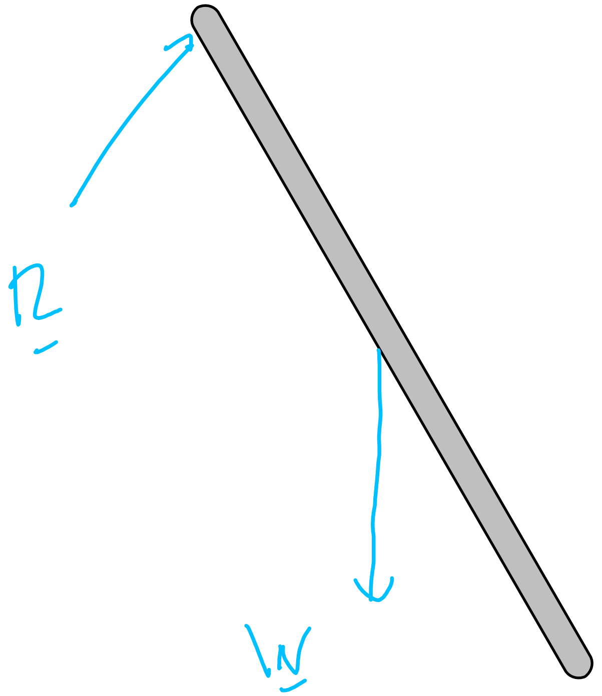
D. Dane Quinn, PhD

Department of Mechanical Engineering
The University of Akron
Akron OH 44325–3903 USA

quinn@uakron.edu
www.uakron.edu

The bar of mass m and length ℓ is pinned at O and is released from rest at an angle θ. Find its resulting angular acceleration at the instant it is released, as well as the reaction force at the pin.





FORCES

- WEIGHT ($\underline{W}$)
- REACTION AT O ($\underline{R}$)

Coordinates and Directions

$$\hat{i} = C_\theta \hat{e}_1 - S_\theta \hat{e}_2 \qquad \hat{e}_1 = C_\theta \hat{i} + S_\theta \hat{j}$$

$$\hat{j} = S_\theta \hat{e}_1 + C_\theta \hat{e}_2 \qquad \hat{e}_2 = -S_\theta \hat{i} + C_\theta \hat{j}$$

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Kinematics

ANGULAR ACCELERATION $\quad \underline{\alpha}_B = \ddot{\theta}\hat{k} \qquad \underline{\omega}_\beta = \dot{\theta}\hat{k}$

$$\underline{a}_G = \underline{a}_O + \underline{\alpha}_B \times \underline{r}_{G/O} + \underline{\omega}_\beta \times (\underline{\omega}_\beta \times \underline{r}_{G/O}) \qquad \underline{r}_{G/O} = -\frac{l}{2}\hat{e}_2$$

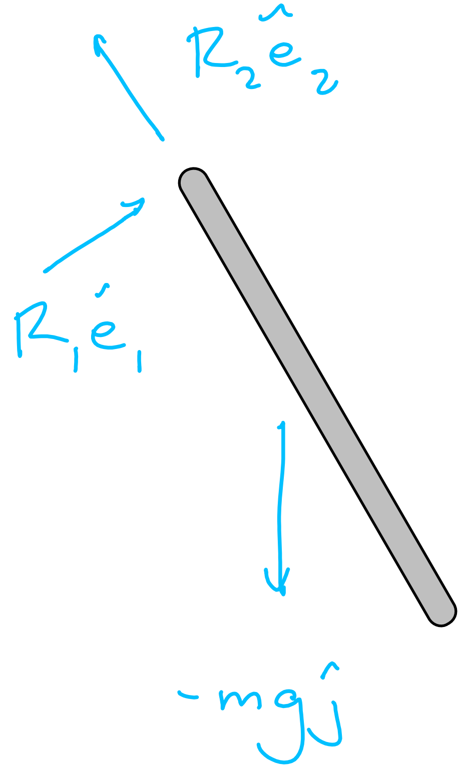
$$= \underline{0} + (\ddot{\theta}\hat{k}) \times \left(-\frac{l}{2}\hat{e}_2\right) + (\dot{\theta}\hat{k}) \times \left((\dot{\theta}\hat{k}) \times \left(-\frac{l}{2}\hat{e}_2\right)\right)$$

$$= \frac{l\ddot{\theta}}{2}\hat{e}_1 + \frac{l\dot{\theta}^2}{2}\hat{e}_2 \qquad @\ t=0 \quad \underline{\omega}_B = \underline{0}$$

Free Body Diagram

REACTION FORCE

$$\underline{R} = R_1\hat{e}_1 + R_2\hat{e}_2$$



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Equations of Motion

LINEAR MOMENTUM BALANCE

$$\sum \underline{F} = R_1 \hat{e}_1 + R_2 \hat{e}_2 - mg\hat{j} = \frac{ml\ddot{\theta}}{2}\hat{e}_1 + \frac{ml\dot{\theta}^2}{2}\hat{e}_2 = m\underline{a}_G$$

$$\hat{j} = S_\theta \hat{e}_1 + C_\theta \hat{e}_2$$

$\hat{e}_1$ DIR: $$R_1 - mgS_\theta = \frac{ml\ddot{\theta}}{2}$$

$\hat{e}_2$ DIR: $$R_2 - mgC_\theta = \frac{ml\dot{\theta}^2}{2}$$

ANGULAR MOMENTUM BALANCE

MOMENT ABOUT O FROM REACTION VANISHE

$$\underline{M}_{GRAV} = \underline{r}_{G/O} \times \underline{W} = \left(-\frac{l}{2}\hat{e}_2\right) \times \left(-mg\left(S_\theta \hat{e}_1 + C_\theta \hat{e}_2\right)\right) = -\frac{mglS_\theta}{2}\hat{k}$$

$$\sum \underline{M}_O = -\frac{mgl}{2}S_\theta \hat{k} = \frac{ml^2}{3}\ddot{\theta}\hat{k} = \underline{\underline{I}}^B_O \underline{\alpha}_B$$

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TAKING COMPONENTS IN THE $\hat{k}$ DIRECTION

$$-\frac{mgl}{2} S_\theta = \frac{ml^2}{3}\ddot{\theta} \longrightarrow \ddot{\theta} = -\frac{3g}{2l} S_\theta$$

$$\alpha_B = \ddot{\theta}\hat{k} = \left(-\frac{3g}{2l} S_\theta\right)\hat{k}$$

REACTION FORCE

$$R_1 = mg S_\theta + \frac{ml\ddot{\theta}}{2}$$

$$R_2 = mg C_\theta + \frac{ml\dot{\theta}^2}{2}$$

$$\longrightarrow$$

$$R_1 = mg S_\theta - \frac{3mg}{4} S_\theta = \frac{mg}{4} S_\theta$$

$$R_2 = mg C_\theta$$

$$\underline{R} = R_1\hat{e}_1 + R_2\hat{e}_2 = \left[\frac{mg}{4} S_\theta\right]\hat{e}_1 + \left[mg C_\theta\right]\hat{e}_2$$

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