Planar Dynamics

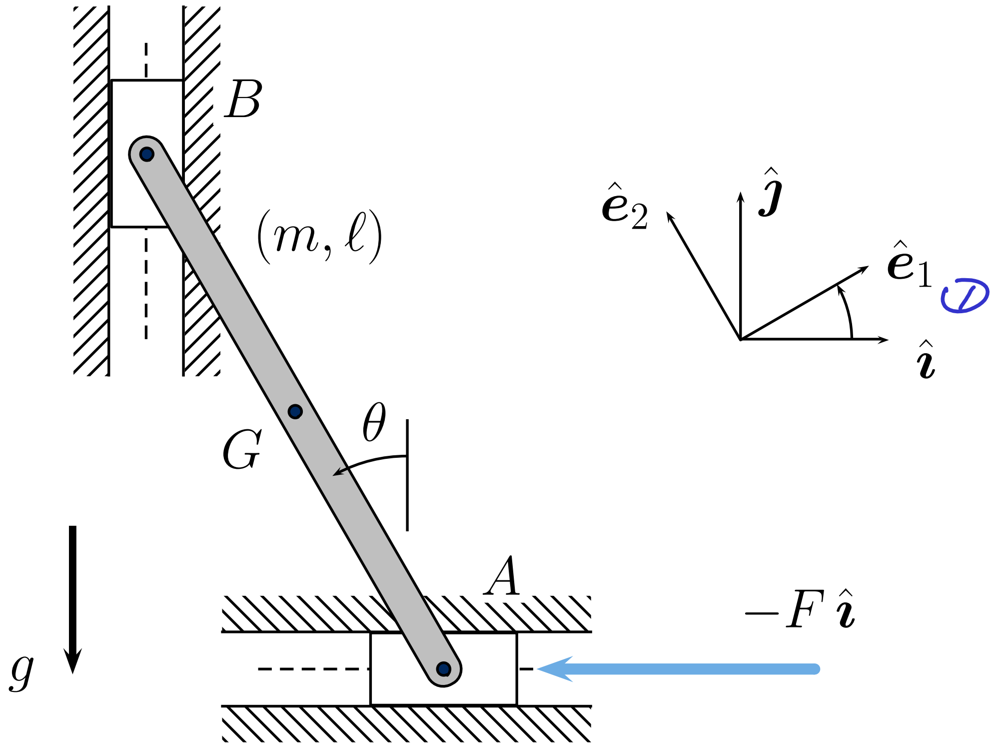
Engineering Mechanics: Dynamics

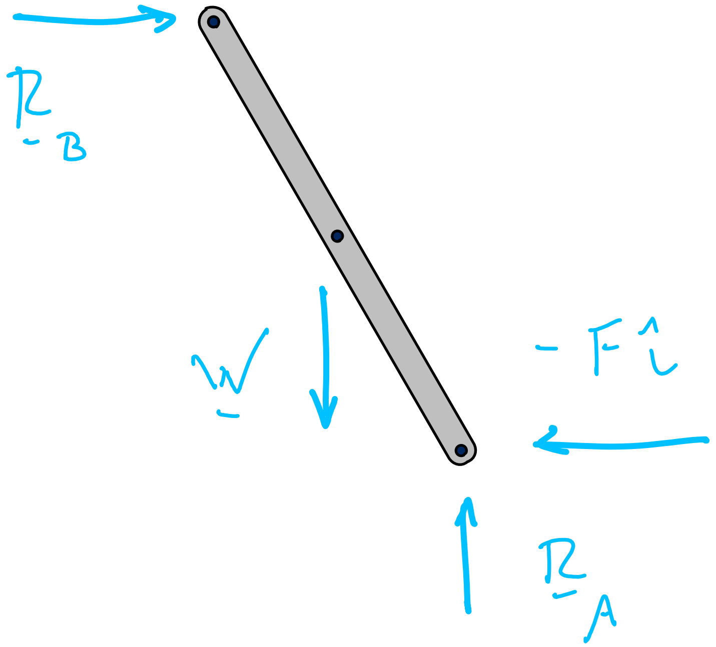
D. Dane Quinn, PhD

Department of Mechanical Engineering
The University of Akron
Akron OH 44325–3903 USA

quinn@uakron.edu
www.uakron.edu

The bar of mass m and length ℓ slides along the smooth horizontal and vertical slots and the (massless) block at A is subject to a force $-F\,\hat{\boldsymbol{\imath}}$. Find the resulting angular acceleration of the bar if it is released from rest at angle θ.





$\underline{W}$: WEIGHT

$\underline{R}$: REACTION FORCES

$-F\hat{\imath}$: EXTERNAL FORCES

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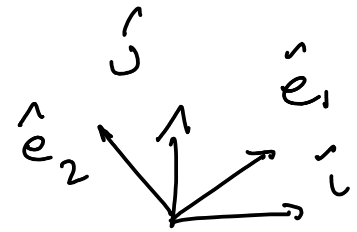
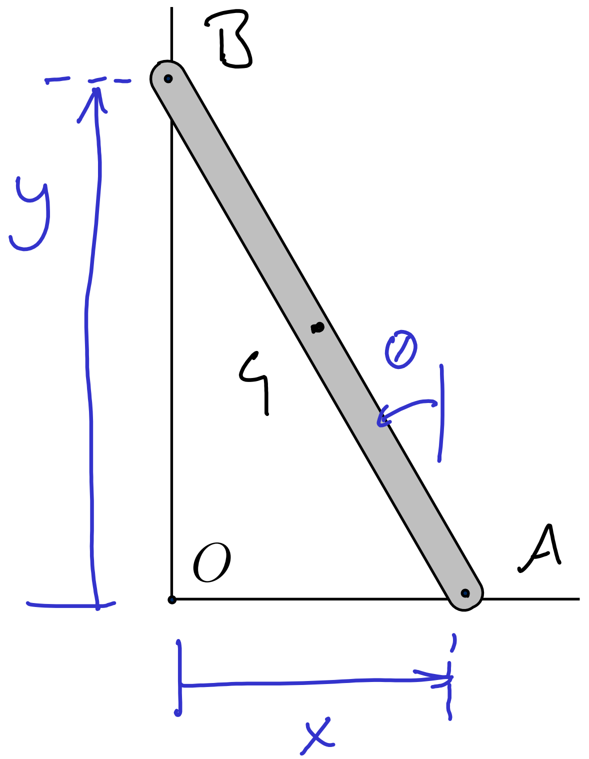
Coordinates and Directions

$$\hat{e}_1 = C_\theta \hat{\imath} + S_\theta \hat{\jmath} \qquad \hat{e}_2 = -S_\theta \hat{\imath} + C_\theta \hat{\jmath}$$





$$\underline{a}_A = \ddot{x}\,\hat{\imath} \qquad \underline{a}_B = \ddot{y}\,\hat{\jmath}$$

Kinematics

$$\underline{a}_B = \underline{a}_A + \underline{\alpha}_\beta \times \underline{r}_{B/A} + \cancelto{0}{\underline{\omega}_\beta \times \left(\underline{\omega}_\beta \times \underline{r}_{B/A}\right)}$$

$\underline{\omega}_\beta = \underline{0}$ AT THIS INSTANT

$$\ddot{y}\,\hat{\jmath} = \ddot{x}\,\hat{\imath} + (\ddot{\theta}\,\hat{k}) \times (\ell\,\hat{e}_2)$$

$$= \ddot{x}\,\hat{\imath} - \ell\ddot{\theta}\,\hat{e}_1 = \left[\ddot{x} - \ell\ddot{\theta} C_\theta\right]\hat{\imath} + \left[-\ell\ddot{\theta} S_\theta\right]\hat{\jmath}$$

$$\ddot{x} = \ell\ddot{\theta} C_\theta \quad , \quad \ddot{y} = -\ell\ddot{\theta} S_\theta$$

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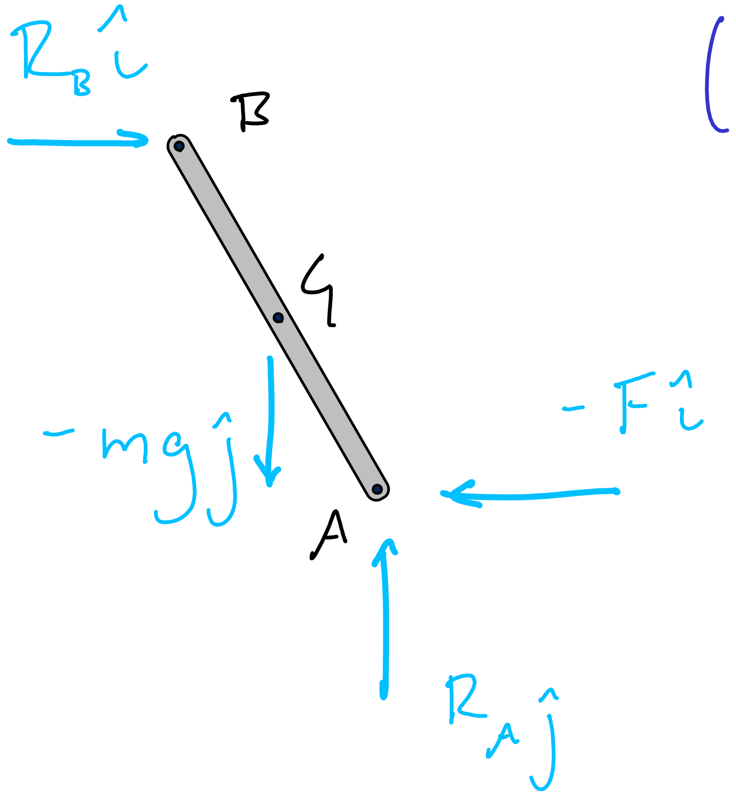
$$\underline{a}_G = \underline{a}_A + \underline{\alpha}_B \times \underline{r}_{G/A} + \underline{\omega}_B \times \cancel{\left(\underline{\omega}_B \times \underline{r}_{G/A}\right)}^{\,0}$$

$$= \ddot{x}\,\hat{\imath} + \left(\ddot{\theta}\hat{k}\right) \times \left(\frac{l}{2}\hat{e}_2\right)$$

$$= \left(l\ddot{\theta}\,c_\theta\right)\hat{\imath} - \frac{l\ddot{\theta}}{2}\hat{e}_1 = \left[\frac{l\ddot{\theta}}{2}c_\theta\right]\hat{\imath} + \left[-\frac{l\ddot{\theta}}{2}s_\theta\right]\hat{\jmath}$$

Free Body Diagram/Equations of Motion



$$\left(R_B - F\right)\hat{\imath} + \left(R_A - mg\right)\hat{\jmath} = \sum \underline{F} = m\underline{a}_G = m\left\{\left[\frac{l\ddot{\theta}}{2}c_\theta\right]\hat{\imath} + \left[-\frac{l\ddot{\theta}}{2}s_\theta\right]\hat{\jmath}\right\}$$

$$\left(R_A\frac{l}{2}s_\theta - \left(R_B + F\right)\frac{l}{2}c_\theta\right)\hat{k} = \sum \underline{M}_G = I_G\underline{\alpha}_B = \frac{ml^2}{12}\left(\ddot{\theta}\hat{k}\right)$$

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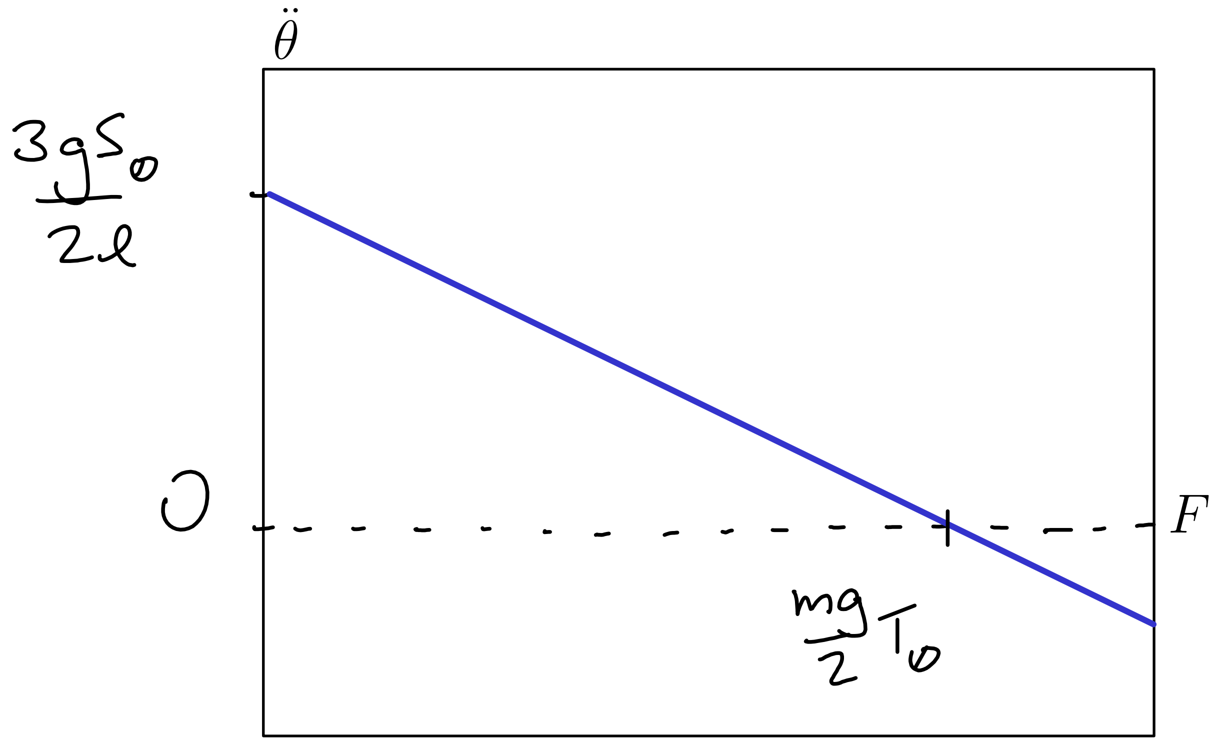
$$R_B - F = \frac{m\ell\ddot{\theta}C_\theta}{2}, \qquad R_A - mg = -\frac{m\ell\ddot{\theta}S_\theta}{2}, \qquad \frac{\ell S_\theta}{2}R_A - \frac{\ell C_\theta}{2}\left(R_B + F\right) = \frac{m\ell^2}{12}\ddot{\theta}$$

$$\left.\begin{aligned} R_A &= mg - \frac{m\ell S_\theta}{2}\ddot{\theta} \\ R_B &= F + \frac{m\ell C_\theta}{2}\ddot{\theta} \end{aligned}\right\} \longrightarrow \frac{\ell S_\theta}{2}\left(mg - \frac{m\ell S_\theta}{2}\ddot{\theta}\right) - \frac{\ell C_\theta}{2}\left(2F + \frac{m\ell C_\theta}{2}\ddot{\theta}\right) = \frac{m\ell^2}{12}\ddot{\theta}$$

So THAT

$$\ddot{\theta} = \frac{3}{m\ell}\left[\frac{mgS_\theta}{2} - FC_\theta\right]$$



$$R_A = \left(1 - \frac{3S_\theta^2}{4}\right)mg + \frac{3S_\theta C_\theta}{2}F$$

$$R_B = \frac{3S_\theta C_\theta}{4}mg + \left(1 - \frac{3C_\theta^2}{4}\right)F$$

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