Rigid Body Dynamics

Engineering Mechanics: Dynamics

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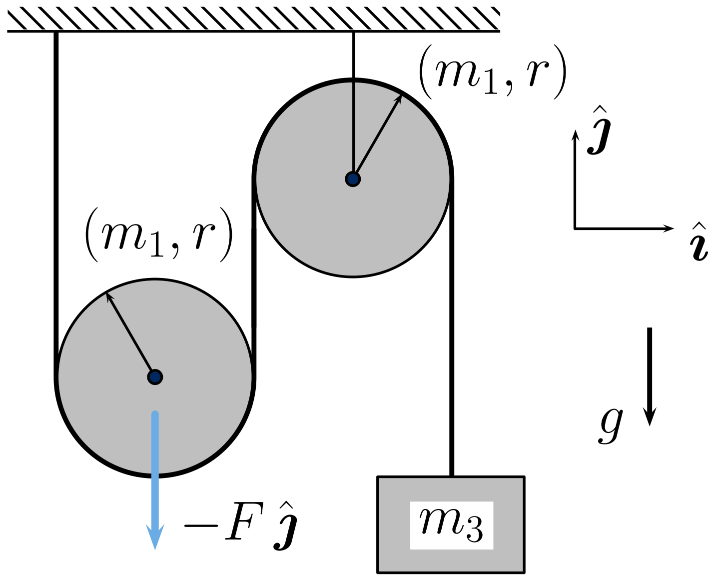
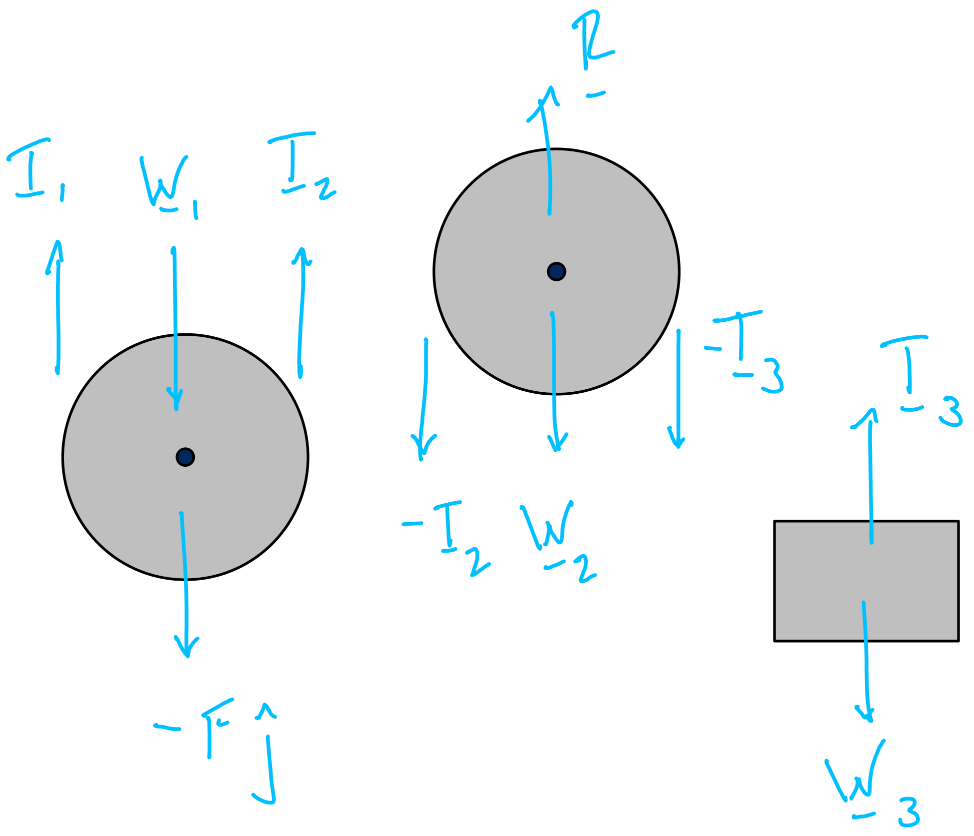
For the pulley system shown, each pulley as mass m_1 and radius r, while the block has mass m_3. If the left pulley is subject to the force $-F\,\hat{\jmath}$ find the resulting acceleration of the block.





TENSION $(\underline{T}_1, \underline{T}_2, \underline{T}_3)$

REACTION FORCE $(\underline{R})$

WEIGHT $(\underline{W}_1, \underline{W}_2, \underline{W}_3)$

EXTERNAL $(-F\hat{\jmath})$

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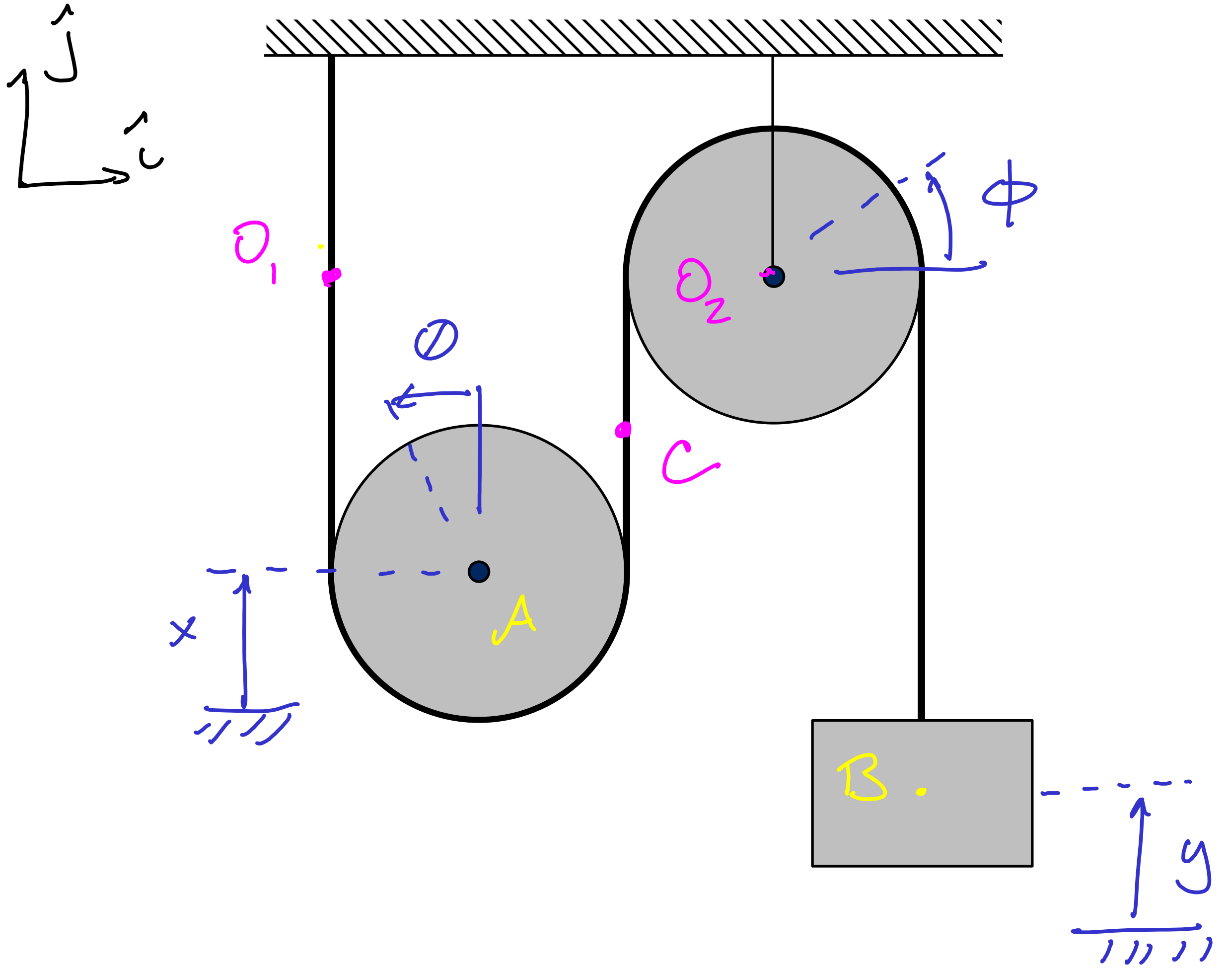
Coordinates and Directions/Kinematics



PULLEY 1

$$\underline{a}_A = \ddot{x}\hat{j} \qquad \underline{\alpha}_1 = \ddot{\theta}\hat{k}$$

PULLEY 2:

$$\underline{a}_{O_2} = \underline{0} \qquad \underline{\alpha}_2 = \ddot{\phi}\hat{k}$$

BLOCK:

$$\underline{a}_B = \ddot{y}\hat{j}$$

$$\underline{v}_A = \frac{\cancelto{\underline{0}}{\underline{v}_{O_1}} + \underline{v}_C}{2}, \qquad \cancelto{\underline{0}}{\underline{v}_{O_2}} = \frac{\underline{v}_C + \underline{v}_B}{2}$$

$$\underline{v}_C = -\underline{v}_B \qquad \underline{v}_A = -\frac{\underline{v}_B}{2}$$

$$\underline{v}_A = \dot{x}\hat{j} = -\frac{\dot{y}\hat{j}}{2} \longrightarrow \dot{x} = -\frac{\dot{y}}{2}$$

LIKEWISE

$$\underline{v}_A = \underline{v}_{O_1} + \underline{\omega}_1 \times \underline{r}_{A/O_1}$$

$$\dot{x}\hat{j} = \underline{0} + \dot{\theta}\hat{k} \times (r\hat{i})$$

$$\dot{x}\hat{j} = r\dot{\theta}\hat{j}$$

$$\downarrow$$

$$\dot{x} = r\dot{\theta}$$

$$\underline{v}_B = \underline{v}_{O_2} + \underline{\omega}_2 \times \underline{r}_{B/O_2}$$

$$\dot{y}\hat{j} = \underline{0} + \dot{\phi}\hat{k} \times (r\hat{i})$$

$$\dot{y}\hat{j} = r\dot{\phi}\hat{j}$$

$$\downarrow$$

$$\dot{y} = r\dot{\phi}$$

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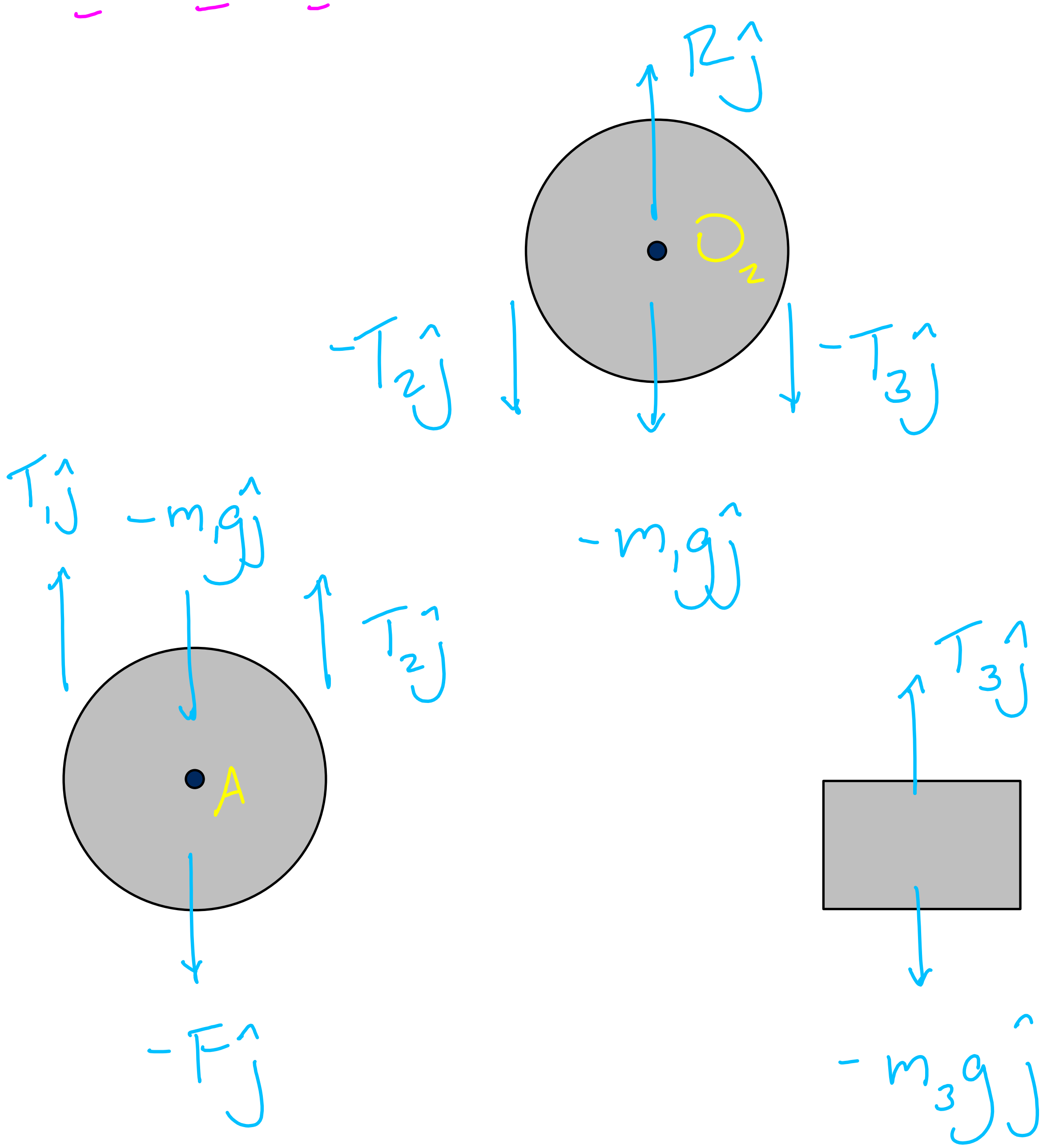
Free Body Diagram/Equations of Motion

$\underline{M} = \underline{r} \times \underline{F}$



MOMENTUM BALANCE

PULLEY 1:

$$\Sigma \underline{F} = T_1\hat{j} - m_1 g\hat{j} + T_2\hat{j} - F\hat{j} = m_1\ddot{x}\hat{j} = m_1 \underline{a}_A$$

$$\Sigma \underline{M}_A = (rT_2 - rT_1)\hat{k} = \frac{m_1 r^2}{2}\ddot{\theta}\hat{k} = I^1_A \underline{\alpha}_1$$

PULLEY 2:

$$\Sigma \underline{F} = -T_2\hat{j} + R\hat{j} - T_3\hat{j} - m_1 g\hat{j} = m_1\underline{0} = m\underline{a}_2$$

$$\Sigma \underline{M}_{O_2} = (T_2 r - T_3 r)\hat{k} = \frac{m_1 r^2}{2}\ddot{\phi}\hat{k} = I^2_{O_2}\underline{\alpha}_2$$

BLOCK:

$$\Sigma \underline{F} = T_3\hat{j} - m_3 g\hat{j} = m_3\ddot{y}\hat{j} = m_3\underline{a}_B$$

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TAKING COMPONENTS

$$T_1 + T_2 - m_1 g - F = m_1 \ddot{x} \qquad R - T_2 - T_3 - m_1 g = 0 \qquad T_3 - m_3 g = m_3 \ddot{y}$$

$$r(T_2 - T_1) = \frac{m_1 r^2}{2}\ddot{\theta} \qquad r(T_2 - T_3) = \frac{m_1 r^2}{2}\ddot{\phi}$$

CONSTRAINT EQUATIONS

$$\dot{y} = -2\dot{x} \qquad \dot{x} = r\dot{\theta} \qquad \dot{y} = r\dot{\phi}$$

$$\ddot{y} = -2\ddot{x} \qquad \ddot{x} = r\ddot{\theta} \qquad \ddot{y} = r\ddot{\phi}$$

ELIMINATE T_1, T_2, T_3

$$m_1 g + F - m_1\ddot{x} + \frac{m_1 r}{2}\ddot{\theta} = 2\left(m_3 g + m_3\ddot{y} + \frac{m_1 r}{2}\ddot{\phi}\right)$$

$$\ddot{y} = \frac{F + (m_1 - 2m_3)g}{4m_1 + 2m_3}$$

SO THAT

$$\boxed{\underline{a}_B = \ddot{y}\hat{j} = \left(\frac{F + (m_1 - 2m_3)g}{4m_1 + 2m_3}\right)\hat{j}}$$

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