Motion: Position, Velocity, Acceleration

Engineering Mechanics: Dynamics

D. Dane Quinn, PhD

Department of Mechanical Engineering
The University of Akron
Akron OH 44325–3903 USA

quinn@uakron.edu
www.uakron.edu

Motion is described by

- Position: location of a point in space with respect to a reference point
- Displacement: change in position over a finite time interval
- Velocity: derivative (rate of change) of position
- Acceleration: derivative (rate of change) of velocity

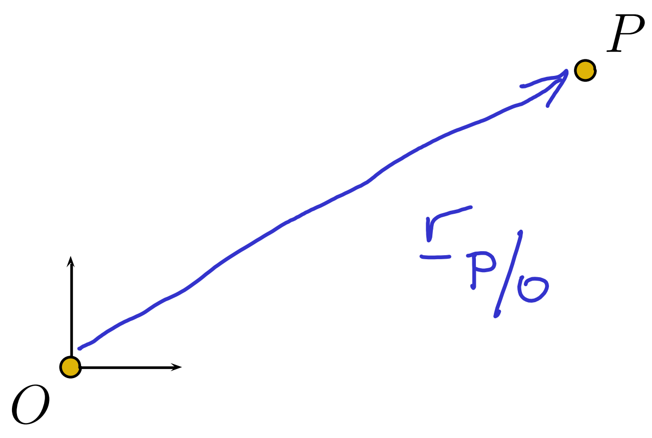
THE MOTION OF A POINT IS DESCRIBED

VECTORS

Position

$\underline{r}_{P/O}$: "THE POSITION OF P WITH RESPECT TO O"

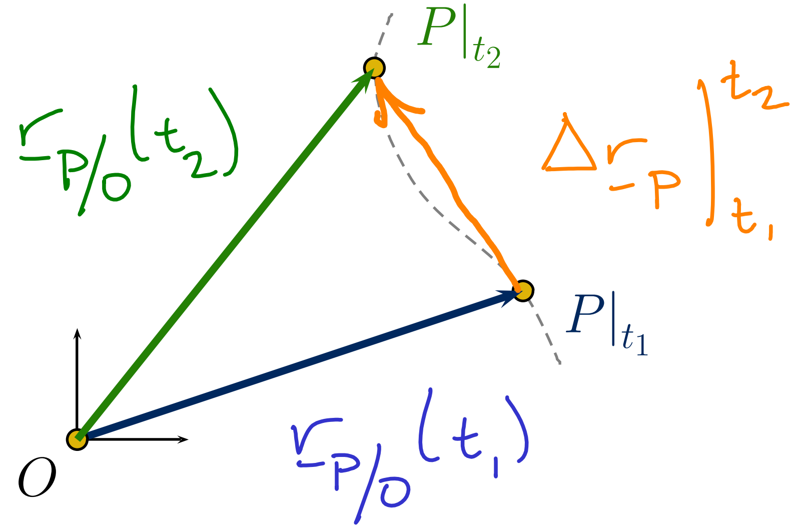
- ORIGIN – POINT OF REFERENCE
- MAGNITUDE – DISTANCE
- DIRECTION – ORIENTION



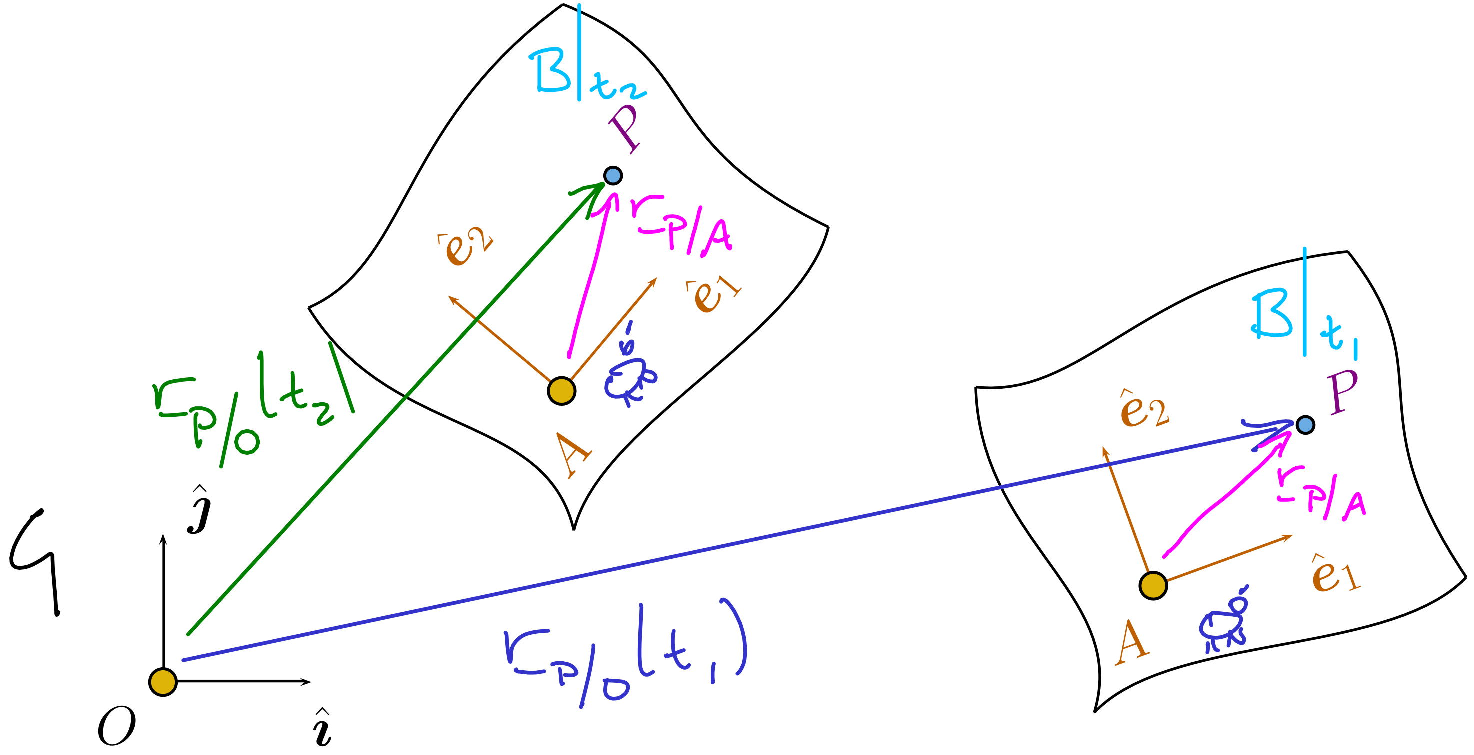
quinn@uakron.edu
www.uakron.edu

Displacement

$$ {}^{F}\Delta \underline{r}_P \Big|_{t_1}^{t_2} = \underline{r}_{P/O}(t_2) - \underline{r}_{P/O}(t_1) $$



THE BACKGROUND (REFERENCE DIRECTIONS) ON WHICH THE CHANGE IS EVALUATED. THE ORIGIN MUST BE FIXED F



$\mathcal{G}$: FIXED IN GROUND; $(\hat{\imath}, \hat{\jmath})$ ARE FIXED IN $\mathcal{G}$

$\mathcal{B}$: MOVING WITH RESPECT TO THE GROUND; $(\hat{e}_1, \hat{e}_2)$ ARE FIXED IN $\mathcal{B}$

THE DISPLACEMENT DEPENDS ON THE FRAME OF REFERENCE

quinn@uakron.edu
www.uakron.edu

Velocity

$${}^{F}\underline{v}_P(t) = \lim_{\Delta t \to 0} \frac{\underline{r}_{P/O}(t+\Delta t) - \underline{r}_{P/O}(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{{}^{F}\Delta \underline{r}_P \Big|_{t}^{t+\Delta t}}{\Delta t} \equiv {}^{F}\frac{d}{dt}\left(\underline{r}_{P/O}\right)$$

DERIVATIVE OF POSITION

- DEPENDS ON THE FRAME OF REFERENCE
- ORIGIN O MUST BE FIXED IN F

– SPEED IS DEFINED AS THE MAGNITUDE OF VELOCITY $v \equiv \|{}^{F}\underline{v}_P\|$

Acceleration

$${}^{F}\underline{a}_P(t) \equiv {}^{F}\frac{d}{dt}\left({}^{F}\underline{v}_P\right) = {}^{F}\frac{d^2}{dt^2}\left(\underline{r}_{P/O}\right)$$

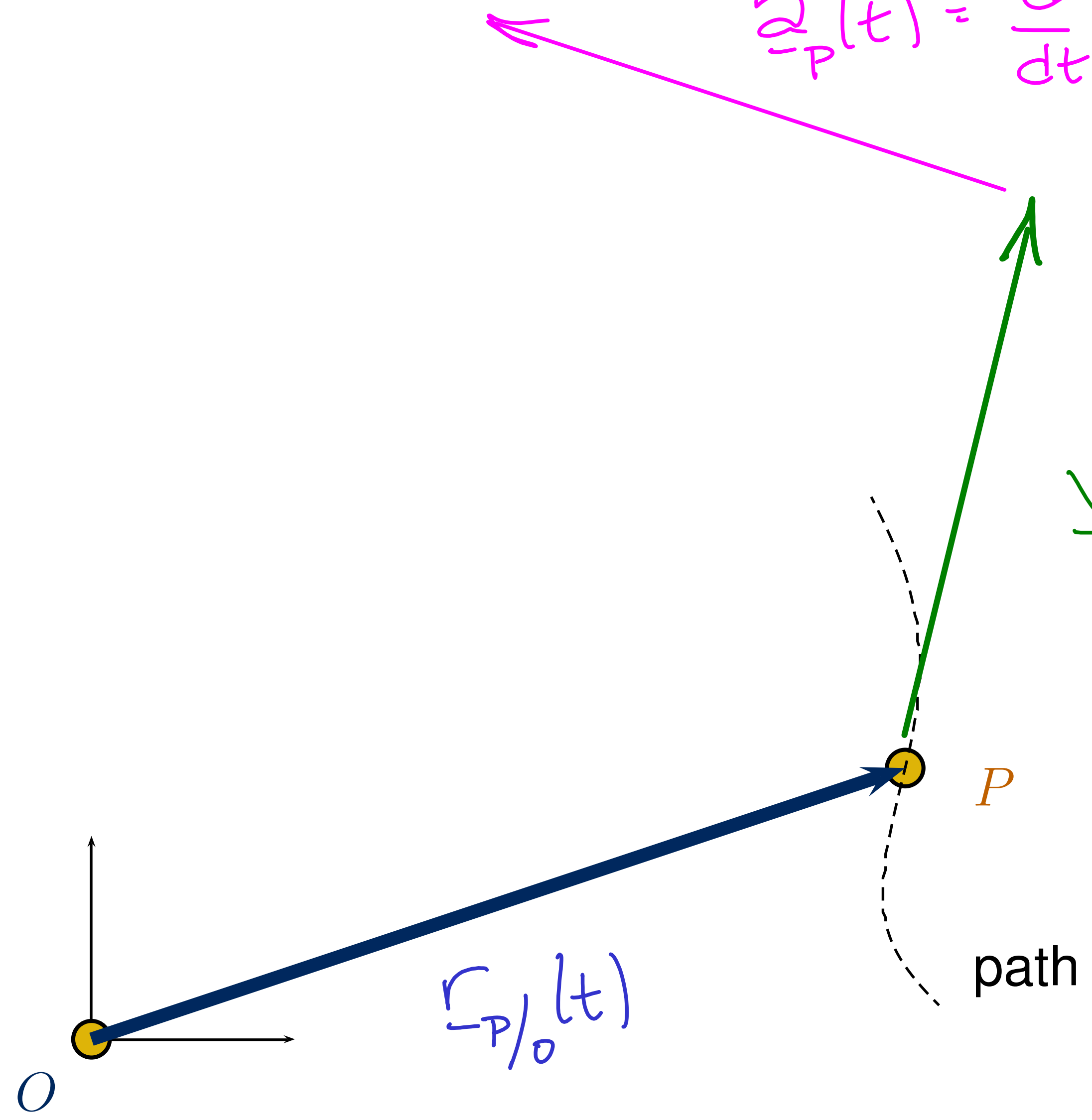
DERIVATIVE OF VELOCITY

- THE FRAME OF REFERENCE FOR DERIVATIVES MUST MATCH

IF NOT WRITTEN $(\underline{v}_P, \underline{a}_P)$, THE FRAME OF REFERENCE IS ASSUMED TO BE THE GROUND (INERTIAL SPACE)

quinn@uakron.edu
www.uakron.edu

$$\underline{a}_P(t) = \frac{d}{dt}\left(\underline{v}_P(t)\right)$$

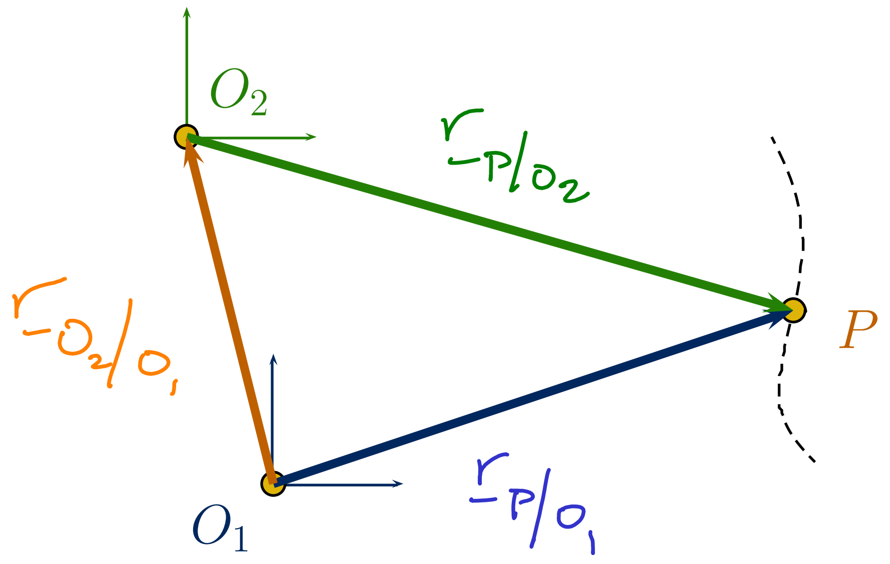


$$\underline{v}_P(t) = \frac{d}{dt}\left(\underline{r}_{P/O}(t)\right)$$

VELOCITY IS TANGENT TO THE PATH

quinn@uakron.edu
www.uakron.edu

THE ORIGIN DOES NOT MATTER AS LONG AS IT IS FIXED THE GROUND



$$\underline{v}_P = \frac{d}{dt}\left(\underline{r}_{P/O_1}\right) = \frac{d}{dt}\left(\underline{r}_{O_2/O_1} + \underline{r}_{P/O_2}\right)$$

$$= \frac{d}{dt}\left(\underline{r}_{O_2/O_1}\right) + \frac{d}{dt}\left(\underline{r}_{P/O_2}\right) \quad \left[\frac{d}{dt}\left(\underline{r}_{O_2/O_1}\right) = \underline{0}\right]$$

$$= \frac{d}{dt}\left(\underline{r}_{P/O_2}\right) = \underline{v}_P$$

$$\underline{a}_P = \frac{d^2}{dt^2}\left(\underline{r}_{P/O_1}\right) = \frac{d^2}{dt^2}\left(\underline{r}_{P/O_2}\right) = \underline{a}_P$$

DIFFERENTIATION $\frac{d}{dt}$ →

Position, $\underline{\mathbf{r}}_{P/O}$	Velocity, $\underline{\mathbf{v}}_P$	Acceleration, $\underline{\mathbf{a}}_P$

← INTEGRATION $\int dt$

quinn@uakron.edu
www.uakron.edu