Motion: Position, Velocity, Acceleration

Engineering Mechanics: Dynamics

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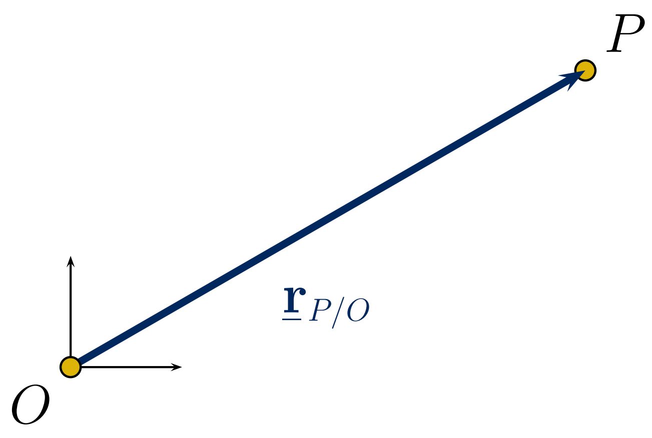
Motion is described by

- Position: location of a point in space with respect to a reference point
- Displacement: change in position over a finite time interval
- Velocity: derivative (rate of change) of position
- Acceleration: derivative (rate of change) of velocity

The motion of a point is naturally described by vectors

Position

$\underline{\mathbf{r}}_{P/O}$: "the position of P with respect to O"



Elements of position

- Origin—point of reference
- Magnitude—distance between the two points
- Direction—relative orientation between the two points

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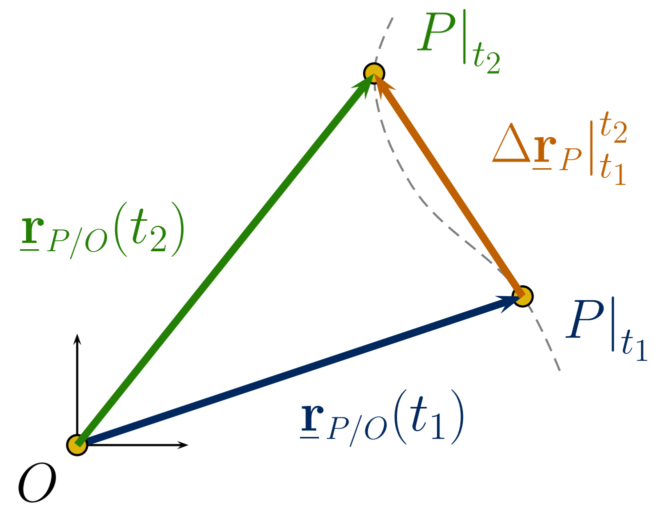
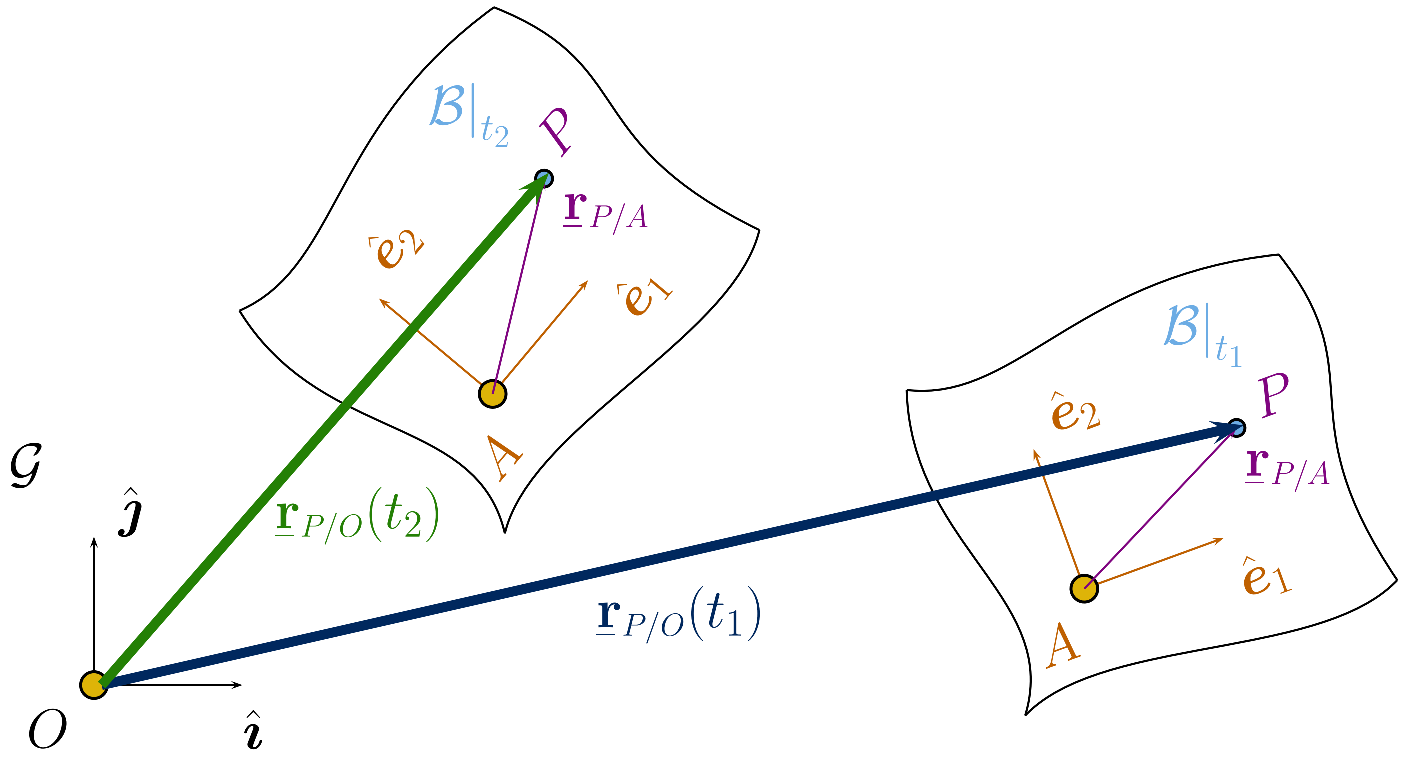
Displacement

Change in position over a finite interval of time

$$^{\mathcal{F}}\Delta\underline{\mathbf{r}}_P\Big|_{t_1}^{t_2} = \underline{\mathbf{r}}_{P/O}(t_2) - \underline{\mathbf{r}}_{P/O}(t_1)$$

The background (reference directions) on which the change is evaluated is the *frame of reference* ($\mathcal{F}$). The origin *must* be fixed in $\mathcal{F}$.





$\mathcal{G}$: fixed in the ground (e.g., inertial space); $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}})$ are fixed with respect to $\mathcal{G}$

$\mathcal{B}$: moving with respect to the ground (e.g., vehicles, merry-go-round); $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2)$ are fixed with respect to $\mathcal{B}$

- The displacement depends on the frame of reference

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Velocity

$$
{}^{\mathcal{F}}\underline{\mathbf{v}}_P(t) = \lim_{\Delta t \to 0} \frac{\underline{\mathbf{r}}_{P/O}(t+\Delta t) - \underline{\mathbf{r}}_{P/O}(t)}{\Delta t},
$$
$$
= \lim_{\Delta t \to 0} \frac{{}^{\mathcal{F}}\Delta \underline{\mathbf{r}}_P \big|_t^{t+\Delta t}}{\Delta t} \equiv {}^{\mathcal{F}}\frac{d}{dt}\left(\underline{\mathbf{r}}_{P/O}\right).
$$

Defined as the derivative of position

- Depends on the frame of reference $\mathcal{F}$
- The origin of the position O must be fixed in the frame of reference $\mathcal{F}$.
- Speed v is the magnitude of velocity, i.e., $v = \|{}^{\mathcal{F}}\underline{\mathbf{v}}_P\|$

Acceleration

$$
{}^{\mathcal{F}}\underline{\mathbf{a}}_P(t) \equiv {}^{\mathcal{F}}\frac{d}{dt}\left({}^{\mathcal{F}}\underline{\mathbf{v}}_P(t)\right) = {}^{\mathcal{F}}\frac{d^2}{dt^2}\left(\underline{\mathbf{r}}_{P/O}\right).
$$

Defined as the derivative of velocity

- Depends on the frame of reference $\mathcal{F}$
- The frame of reference for the velocity must match that for the derivative

Derivatives depend on the frame of reference in which they are taken

If not written (e.g., $\underline{\mathbf{a}}_P$), the frame of reference is assumed to be the ground, i.e., inertial space.

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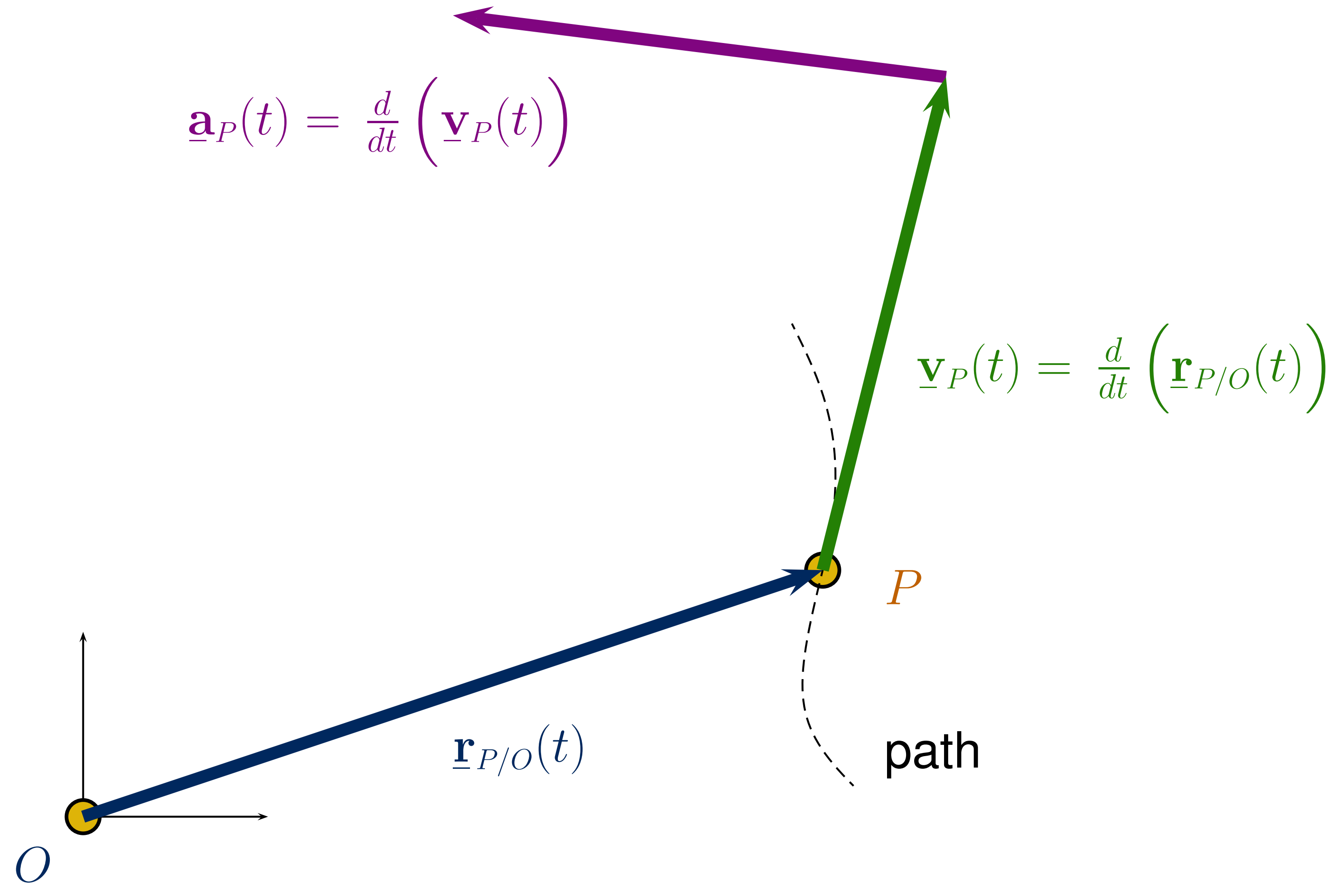
The velocity is tangent to the path



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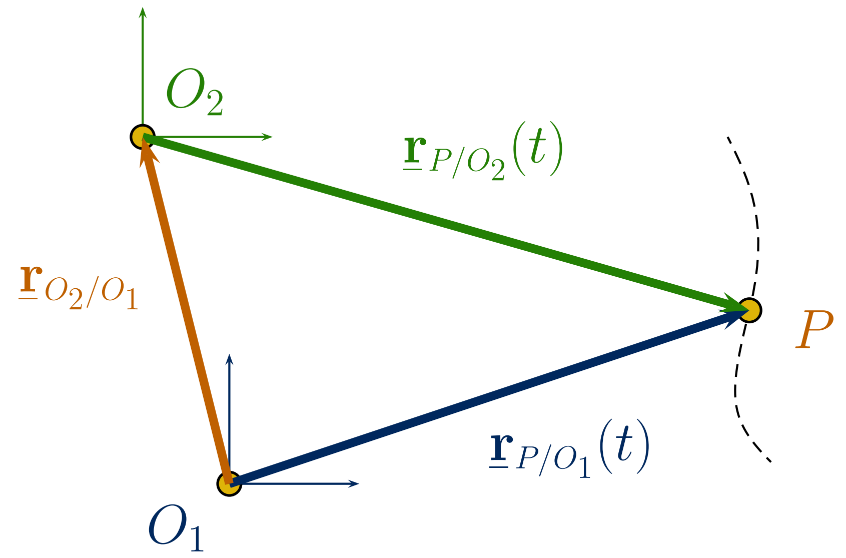
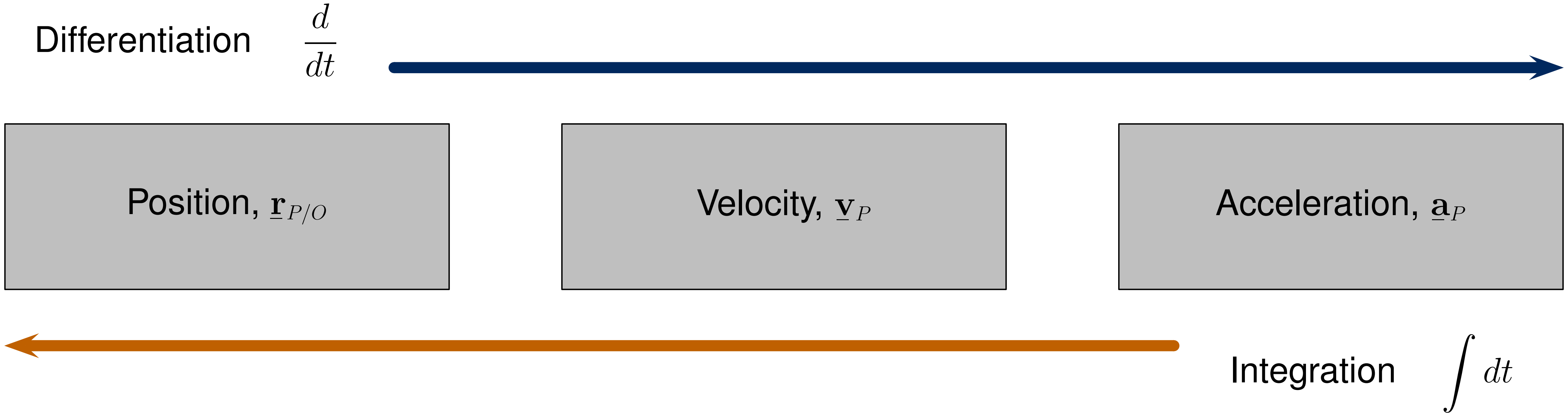
When evaluating the velocity and acceleration the choice of the origin does not matter, provided it is *fixed in the ground*.

$$\underline{\mathbf{v}}_P = \frac{d}{dt}\left(\underline{\mathbf{r}}_{P/O_1}\right) = \frac{d}{dt}\left(\underline{\mathbf{r}}_{O_2/O_1} + \underline{\mathbf{r}}_{P/O_2}\right),$$
$$= \frac{d}{dt}\left(\underline{\mathbf{r}}_{O_2/O_1}\right) + \frac{d}{dt}\left(\underline{\mathbf{r}}_{P/O_2}\right),$$
$$= \frac{d}{dt}\left(\underline{\mathbf{r}}_{P/O_2}\right) = \underline{\mathbf{v}}_P,$$
$$\underline{\mathbf{a}}_P = \frac{d^2}{dt^2}\left(\underline{\mathbf{r}}_{P/O_1}\right) = \frac{d^2}{dt^2}\left(\underline{\mathbf{r}}_{P/O_2}\right) = \underline{\mathbf{a}}_P.$$





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