Particle Dynamics

Engineering Mechanics: Dynamics

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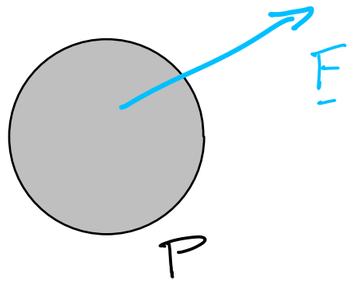
Dynamics: Application of the laws of mechanics to develop equations of motion that describe the response of a system to external effects

Kinematics: Consideration of the allowable motion of an object, consistent with the constraints that act on that object, without regard to the forces that produce motion or the motion that actually occurs

Kinetics: Description of the forces that are applied to a mechanical system



DYNAMICS

$\sum \underline{F} = m \underline{a}_P$

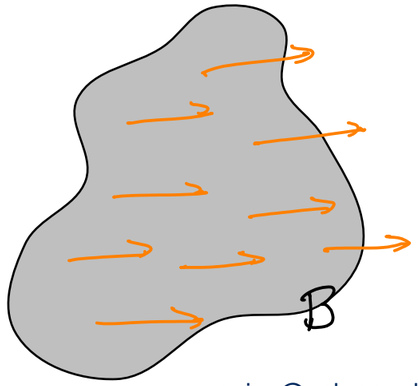
KINETICS KINEMATICS

LINEAR MOMENTUM BALANCE

- VALID ONLY A PARTICLE

IF EVERY POINT ON B UNDERGOES IDENTICAL MOTION (TRANSLATION) THEN B ACTS LIKE A PARTICLE

MORE GENERALLY, OBJECTS CAN TRANSLATE & ROTATE



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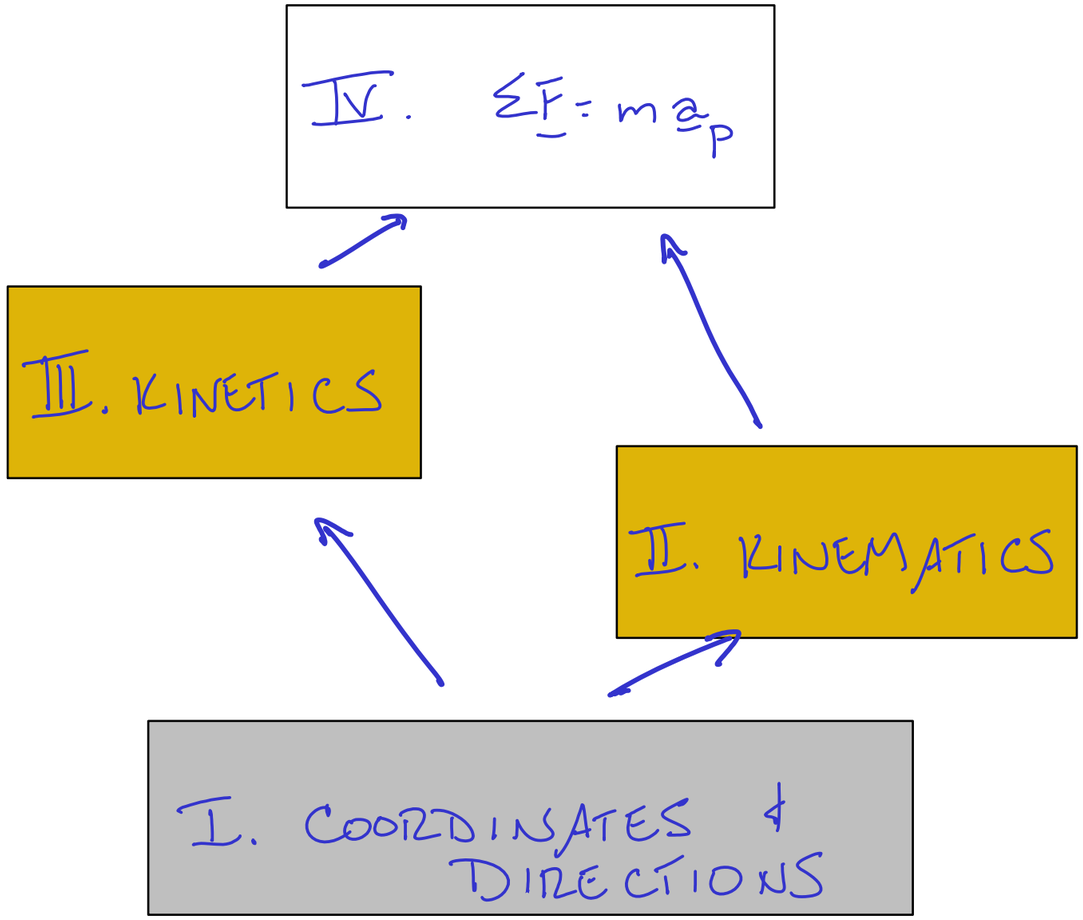

IDENTIFY FORCES & MOTION

DEVELOP EQUATIONS OF MOTION

IV. APPLY LAWS OF MECHANICS

III. DRAW A FREE-BODY DIAGRAM, COMPLETE WITH A VECTOR DESCRIPTION OF ALL FORCES

II. IDENTIFY THE ACCELERATION OF ALL OBJECTS & ANY CONSTRAINTS

I. DEFINE COORDINATES & DIRECTIONS

- IDENTIFY ALL FORCES THAT ACT ON THE SYSTEM & CONSIDER THE POSSIBLE MOTION

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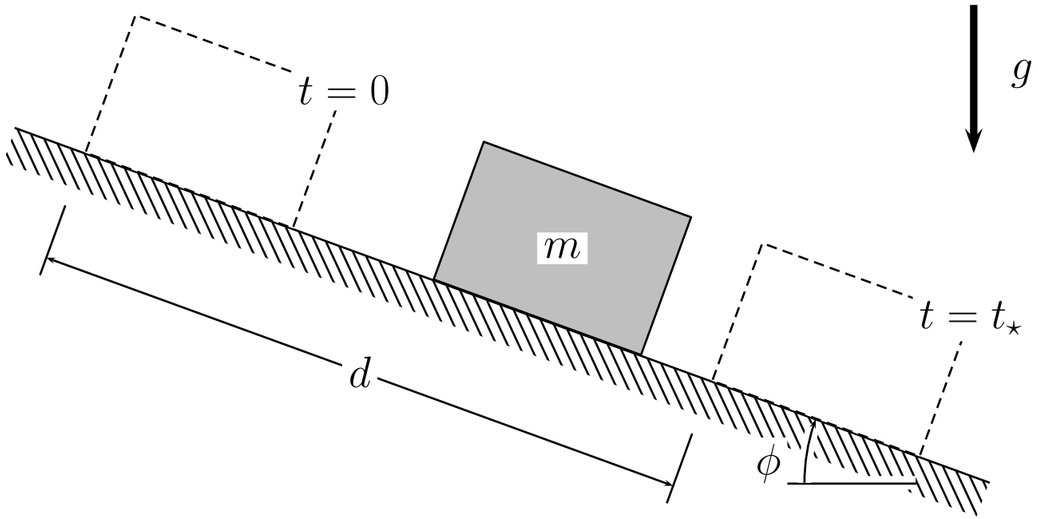
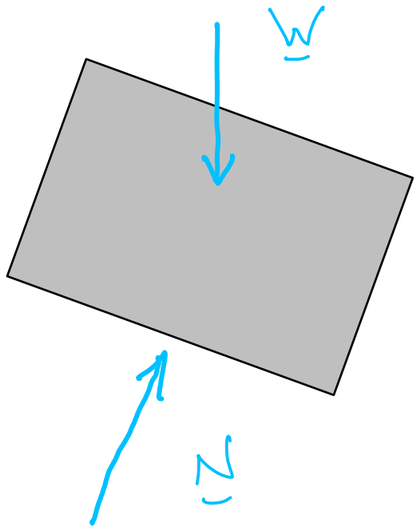
Example

Consider a block of mass m on a smooth plane, inclined at an angle ϕ. Find the acceleration of the block and the time $t_\star$ required to slide a distance d if it is released from rest.



- Think about the problem – IDENTIFY FORCES & CONSIDER THE MOTION



- WEIGHT ($\underline{W}$)
- NORMAL FORCE ($\underline{N}$)

– BLOCK MOVES ALONG THE PLANE

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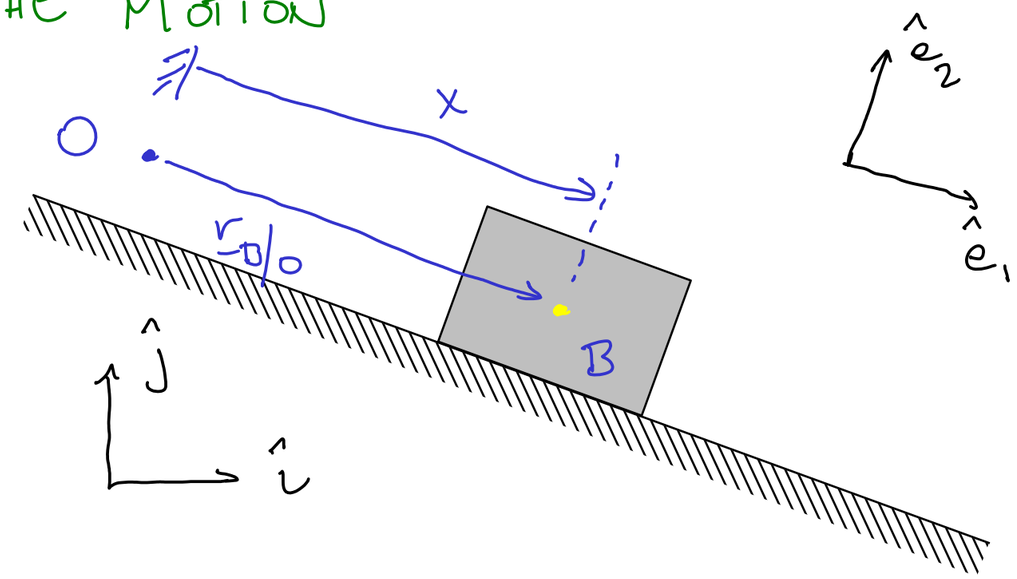
I. Coordinates and Directions

- DEFINE ALL RELEVANT DIRECTIONS, & COORDINATES THAT CAN BE USED TO DESCRIBE THE MOTION

DEFINE $(\hat{i}, \hat{j})$ & $(\hat{e}_1, \hat{e}_2)$

$$\hat{e}_1 = C_\phi \hat{i} - S_\phi \hat{j}$$

$$\hat{e}_2 = S_\phi \hat{i} + C_\phi \hat{j}$$



$$\underline{r}_{B/O} = x \hat{e}_1$$

II. Kinematics

- RELATE THE ACCELERATION TO THE COORDINATES & DIRECTIONS

$$\underline{a}_B = \ddot{x} \hat{e}_1$$

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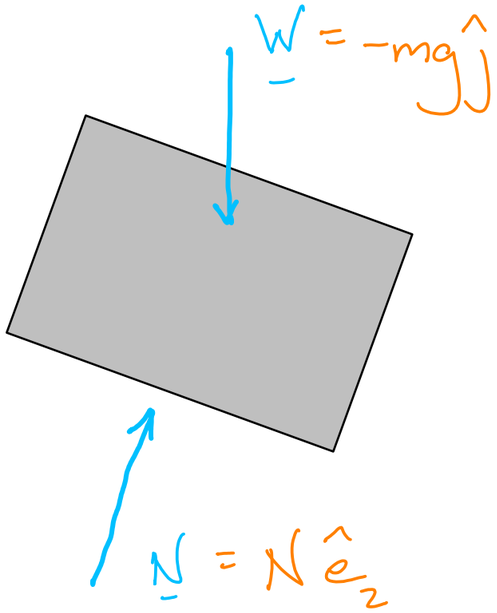
III. Free-body Diagram

- DESCRIBE EACH FORCE AS A VECTOR
- USE THE COORDINATES & DIRECTIONS DEFINED EARLIER



IV. Equations of Motion - APPLY THE LAWS OF MECHANICS

$$N\hat{e}_2 - mg\hat{j} = \sum \underline{F} = m\underline{a}_B = m\ddot{x}\hat{e}_1$$

$$\hat{j} = -S_\phi \hat{e}_1 + C_\phi \hat{e}_2$$

$$N\hat{e}_2 - mg\left(-S_\phi \hat{e}_1 + C_\phi \hat{e}_2\right) = m\ddot{x}\hat{e}_1 + 0\hat{e}_2$$

$\hat{e}_1$ DIR: $\quad mgS_\phi = m\ddot{x}$

$\hat{e}_2$ DIR: $\quad N - mgC_\phi = 0$

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- **Solve** - FIND THE UNKNOWNS REQUESTED BY THE PROBLEM

$$mgS_\phi = m\ddot{x}\,;\quad N - mgC_\phi = 0 \longrightarrow \ddot{x} = gS_\phi\,;\quad N = mgC_\phi$$

ACCELERATION

$$\underline{a}_B = \ddot{x}\,\hat{e}_1 = (gS_\phi)\,\hat{e}_1$$

$$\int_0^t \{\ddot{x} = gS_\phi\}\,d\eta \longrightarrow \dot{x}(t) - \cancel{\dot{x}(0)}^{\,0} = gS_\phi \cdot t$$

$$\int_0^t \{\dot{x} = gS_\phi \cdot \tau\}\,d\tau \longrightarrow x(t) - \cancel{x(0)}^{\,0} = gS_\phi \cdot \frac{t^2}{2}$$

@ $t = t_*$, $x(t_*) = d$

$$x(t_*) = \frac{gS_\phi}{2}\,t_*^2 = d \longrightarrow t_* = \sqrt{\frac{2d}{gS_\phi}}$$

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