Particle Dynamics

Engineering Mechanics: Dynamics

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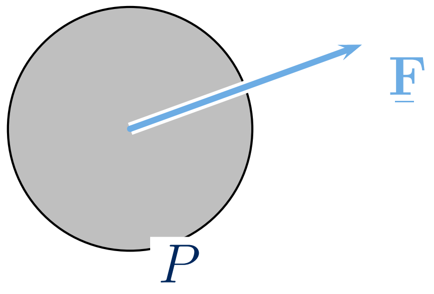
Dynamics: Application of the laws of mechanics to develop equations of motion that describe the response of a system to external effects

Kinematics: Consideration of the allowable motion of an object, consistent with the constraints that act on that object, without regard to the forces that produce motion or the motion that actually occurs

Kinetics: Description of the forces that are applied to a mechanical system



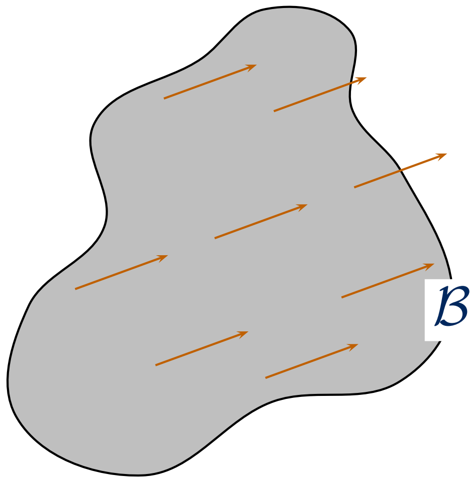
$$\overbrace{\underbrace{\sum \underline{\mathbf{F}}}_{\text{Kinetics}} = \underbrace{m\,\underline{\mathbf{a}}_P}_{\text{Kinematics}}}^{\text{Dynamics}}$$

Linear momentum balance

- Valid for a particle P

If the velocity of every point of a physical object is identical (translation) then the object behaves like a particle.

More generally, objects translate and rotate



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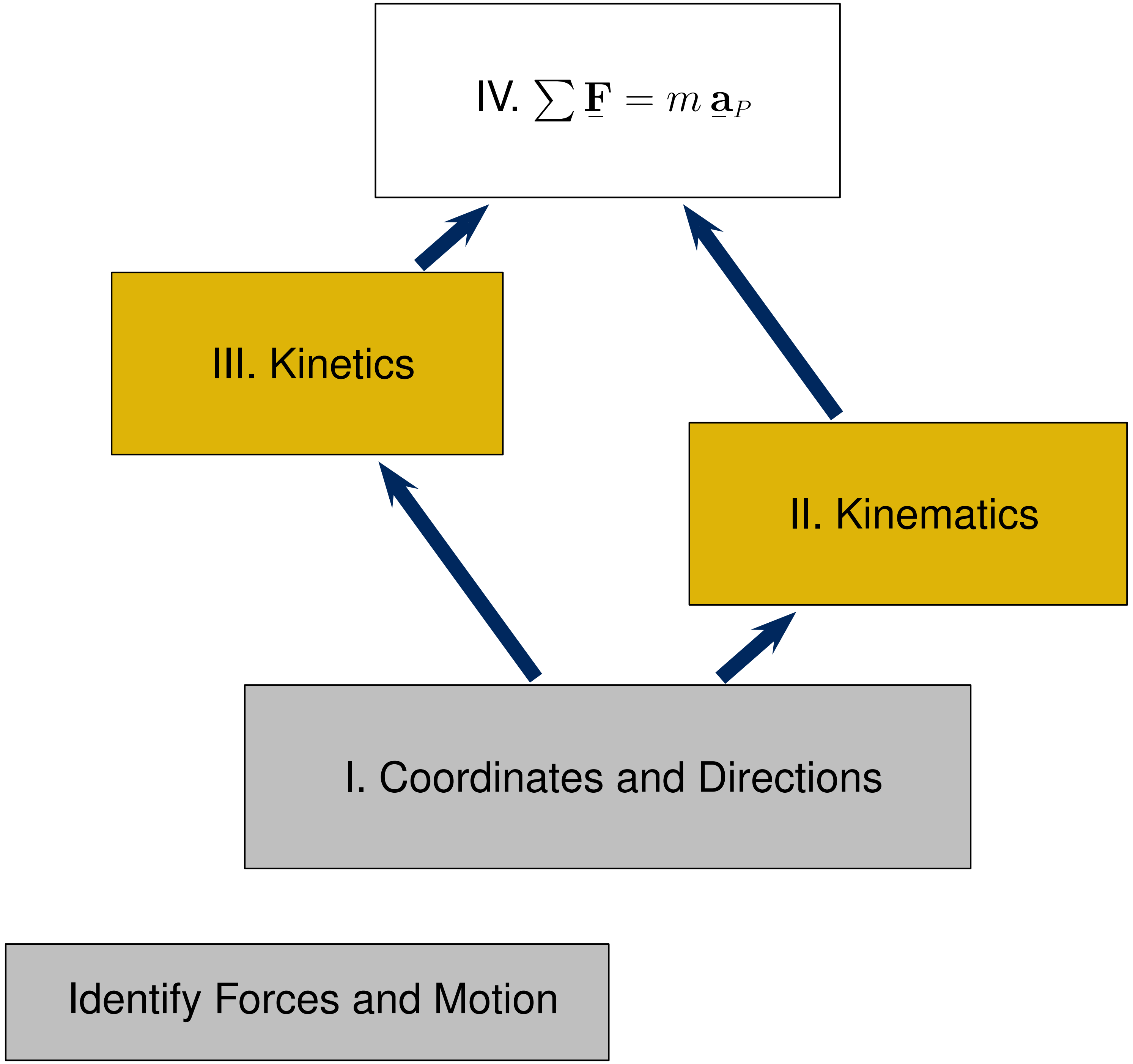

Develop equations of motion

IV. Apply the laws of mechanics

III. Draw a free-body diagram, complete with the vector description of the forces

II. Identify the accelerations of all objects and any constraints on the motion

I. Define coordinates and directions appropriate for the system

- Identify all forces that act on the system and consider the possible configurations of the system

This gives the solution procedure, from bottom to top...

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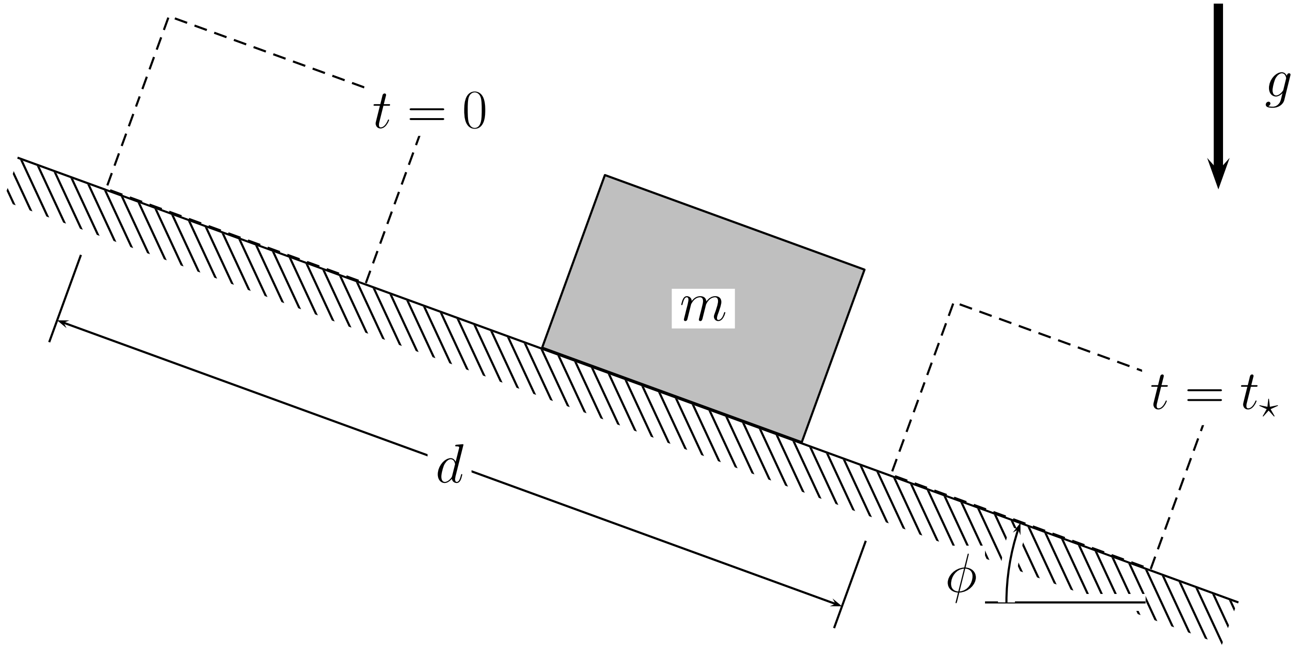
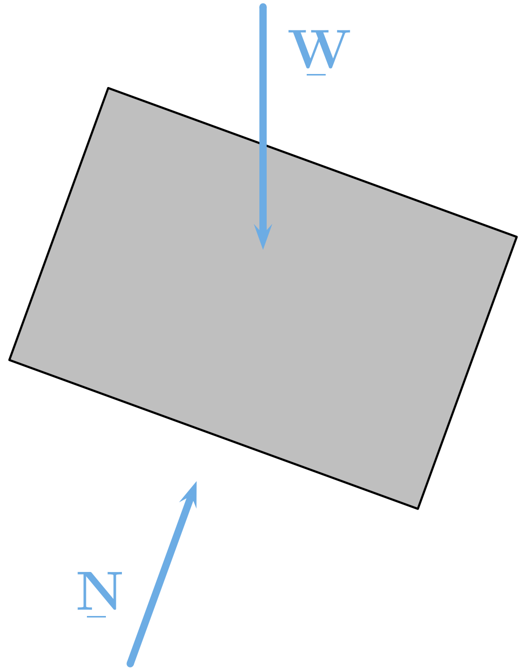
Example

Consider a block of mass m on a smooth plane, inclined at an angle ϕ. Find the acceleration of the block and the time $t_\star$ required to slide a distance d if it is released from rest.



- Think about the problem — identify the forces and consider the motion



We must have an idea of the motion and the forces that act on the system to identify appropriate direction and coordinates for the problem

- The block moves along the inclined plane
- The block is acted on by a normal force $\underline{\mathbf{N}}$ and the gravitational force $\underline{\mathbf{W}}$

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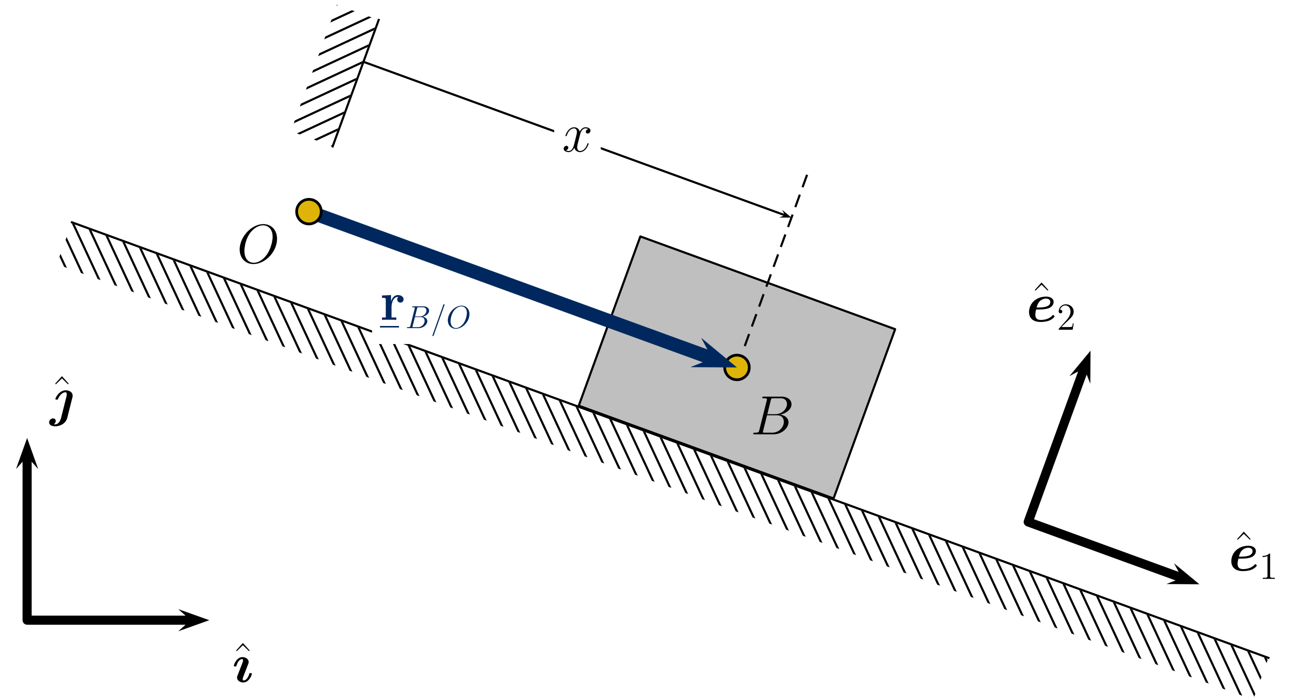
I. Coordinates and Directions — define all directions relevant for the motion and forces, and coordinates that will allow for a description of the motion, in particular the acceleration.

In addition to directions $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}})$, the motion is along the plane

- Define directions $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2)$

The displacement of the block from its starting position can be measured with x, so that

$$\underline{\mathbf{r}}_{B/O} = x\,\hat{\boldsymbol{e}}_1.$$



II. Kinematics — relate the acceleration of the object to the defined coordinates, and account for any constraints.

With these coordinates and directions, the acceleration of the block is

$$\underline{\mathbf{a}}_B = \ddot{x}\,\hat{\boldsymbol{e}}_1.$$

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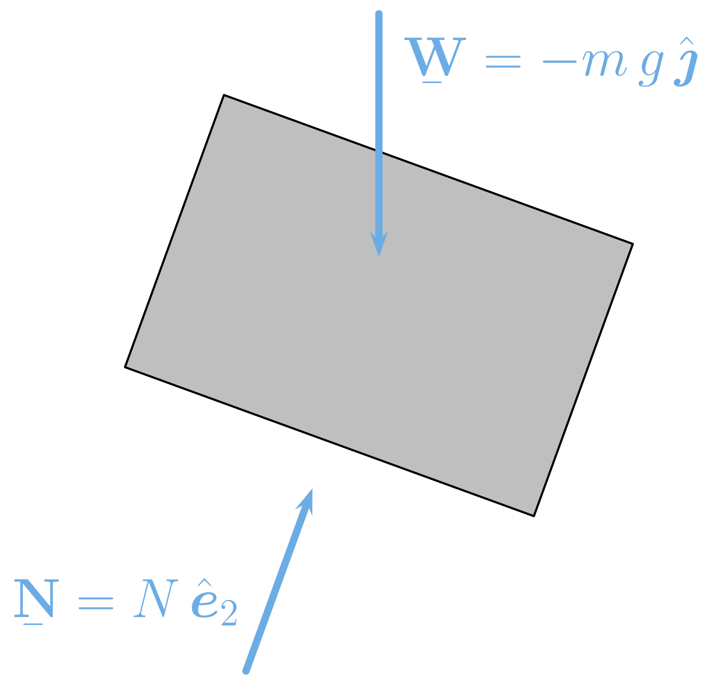
III. Free-body Diagram — a catalog of the forces that act on the system

- From the force diagram earlier, describe each force *completely*—as a vector
- Use the coordinates and directions defined previously



IV. Equations of Motion — apply the laws of mechanics to develop the governing equations of motion for the dynamics.

Apply linear momentum balance to the system

$$N\,\hat{\boldsymbol{e}}_2 - m\,g\,\hat{\boldsymbol{\jmath}} = \sum \underline{\mathbf{F}} = m\,\underline{\mathbf{a}}_B = m\,\ddot{x}\,\hat{\boldsymbol{e}}_1.$$

With $\hat{\boldsymbol{\jmath}} = -\,\mathrm{S}_\phi\;\hat{\boldsymbol{e}}_1 + \mathrm{C}_\phi\;\hat{\boldsymbol{e}}_2$,

$$N\,\hat{\boldsymbol{e}}_2 - m\,g\,\left(-\,\mathrm{S}_\phi\;\hat{\boldsymbol{e}}_1 + \mathrm{C}_\phi\;\hat{\boldsymbol{e}}_2\right) = m\,\ddot{x}\,\hat{\boldsymbol{e}}_1, \quad \longrightarrow \quad \begin{aligned} \hat{\boldsymbol{e}}_1 :& \quad m\,g\,\mathrm{S}_\phi = m\,\ddot{x}, \\ \hat{\boldsymbol{e}}_2 :& \quad N - m\,g\,\mathrm{C}_\phi = 0. \end{aligned}$$

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- Solve — find the unknowns requested in the problem statement.

From the equations of motion

$$\ddot{x} = g\, \mathrm{S}_\phi, \qquad N = m\, g\, \mathrm{C}_\phi,$$

so that acceleration of the block is

$$\underline{\mathbf{a}}_B = g\, \mathrm{S}_\phi\, \hat{\boldsymbol{e}}_1.$$

The block is released from rest and the displacement is measured from the initial position, so that $\dot{x}(0) = 0$, $x(0) = 0$. Integrating the displacement

$$\int_0^t \left\{ \ddot{x} = g\, \mathrm{S}_\phi \right\} d\xi \quad \longrightarrow \quad \dot{x}(t) - \dot{x}(0) = g\, \mathrm{S}_\phi\, t,$$

$$\int_0^t \left\{ \dot{x} = g\, \mathrm{S}_\phi\, \tau \right\} d\tau \quad \longrightarrow \quad x(t) - x(0) = \frac{g\, \mathrm{S}_\phi}{2} t^2.$$

At (unknown) time $t_\star$ the block has traveled a distance d, so that

$$x(t_\star) = d = \frac{g\, \mathrm{S}_\phi}{2} t_\star^2, \quad \longrightarrow \quad t_\star = \sqrt{\frac{2\, d}{g\, \mathrm{S}_\phi}}.$$

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