Planar Kinematics

Engineering Mechanics: Dynamics

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Kinematics: Consideration of the **allowable motion** of an object, consistent with the constraints that act on that object, without regard to the forces that produce motion or the motion that actually occurs

Rigid bodies

- NO INTERNAL DEFORMATION
- RELATIVE DISTANCES ARE CONSTANT

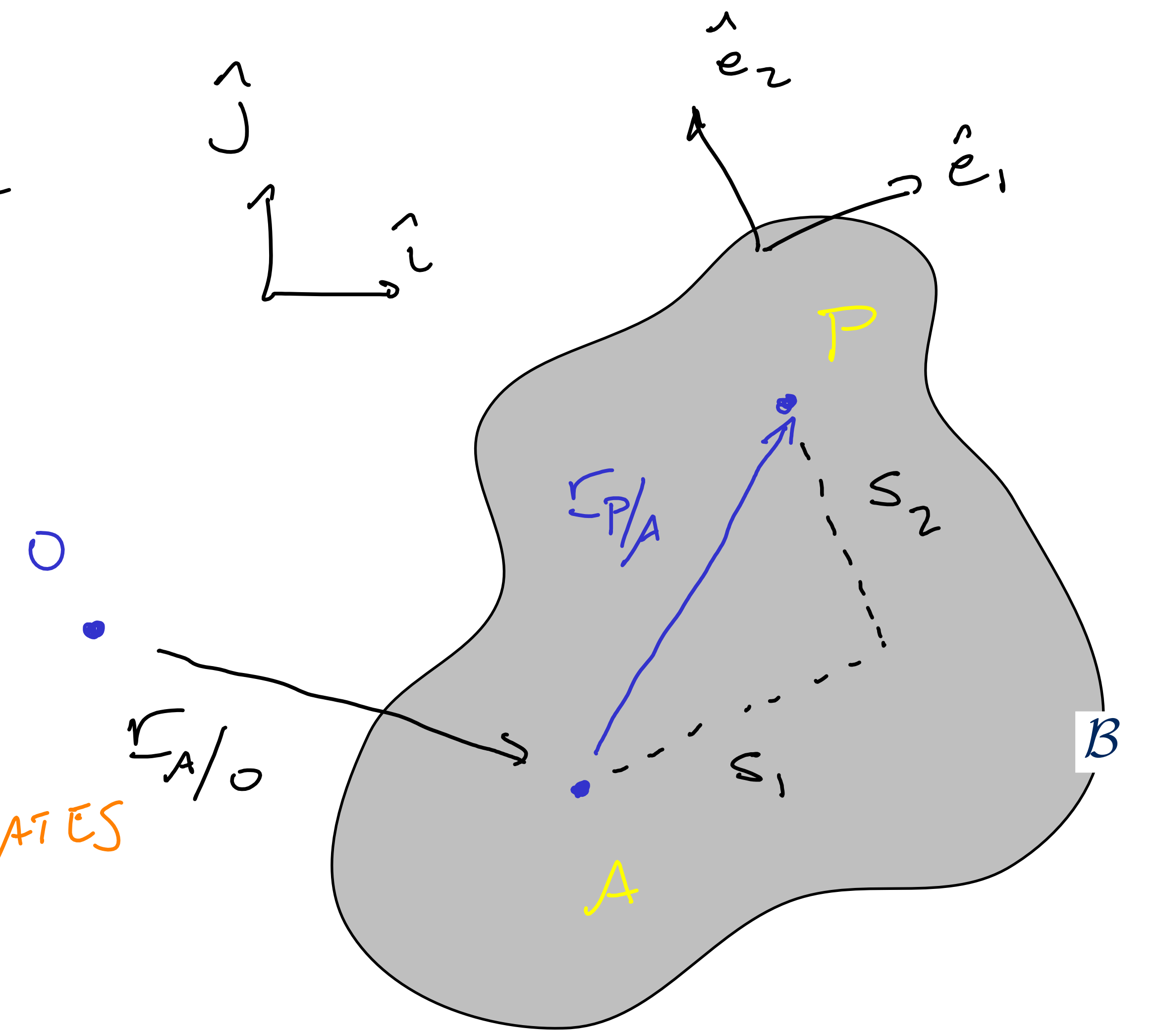
$$r_{P/A} = s_1 \hat{e}_1 + s_2 \hat{e}_2$$

CONSTANT

CONFIGURATION OF B ...

- POSITION OF A $(r_{A/O})$ 2 COORDINATES
- ROTATION OF B
  ORIENTATION OF $(\hat{e}_1, \hat{e}_2)$

(ω_B) 1 COORDINATES



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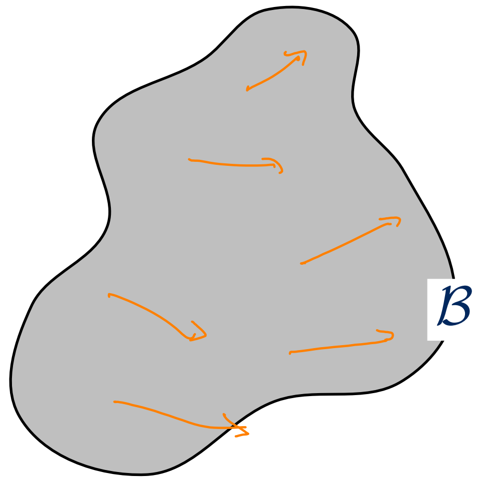


Objects can both translate and rotate, so that each point of a rigid object has a *different velocity and acceleration*.

EXPRESS

MOTION OF P ($\underline{r}_{P/O}, \underline{v}_P, \underline{a}_P$)

IN TERMS OF

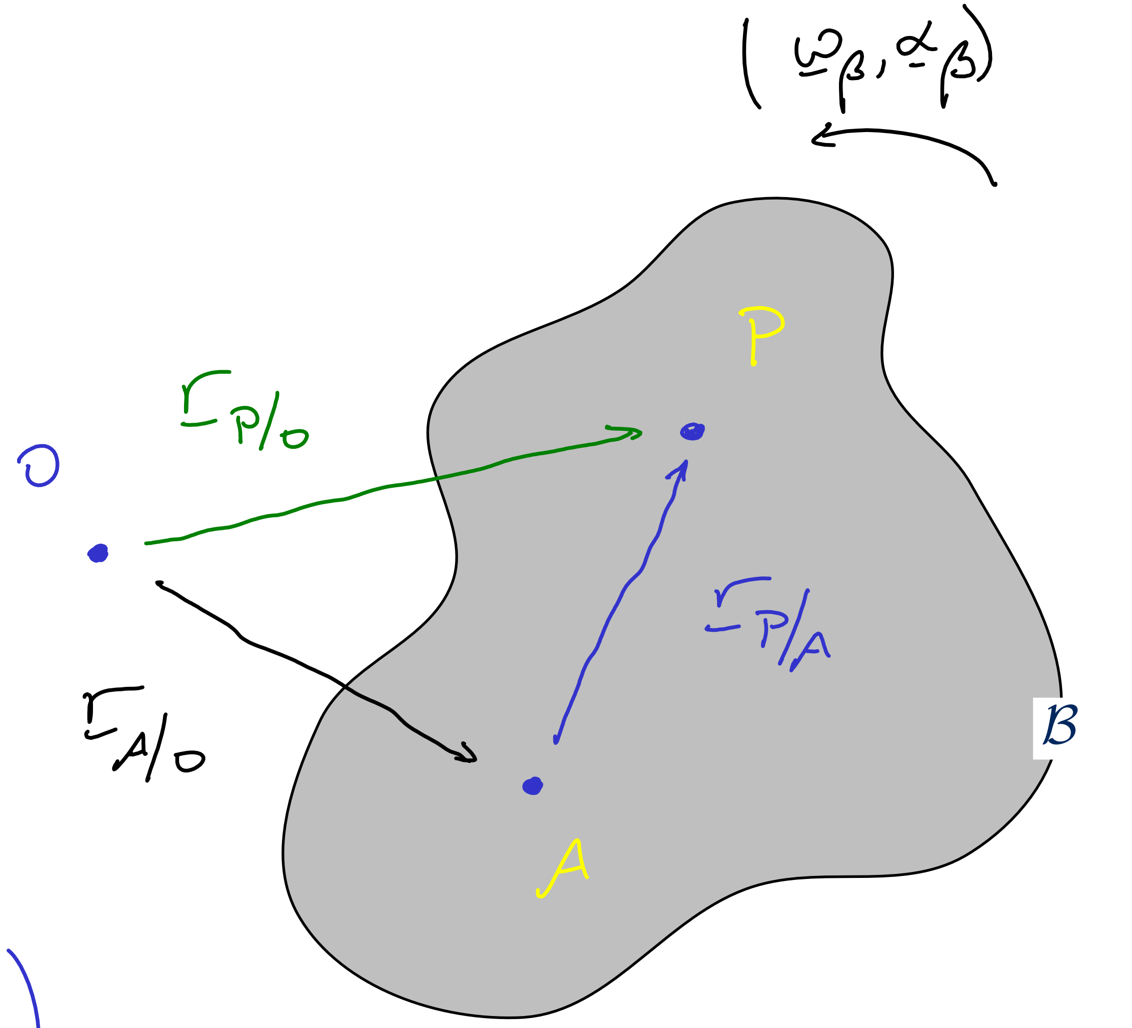
MOTION OF A ($\underline{r}_{A/O}, \underline{v}_A, \underline{a}_A$)

& ROTATION OF $\mathcal{B}$ ($\underline{\omega}_{\mathcal{B}}, \underline{\alpha}_{\mathcal{B}}$)

TOGETHER WITH

RELATIVE MOTION OF P

WITH RESPECT TO A ($\underline{r}_{P/A}, {}^{\mathcal{B}}\underline{v}_P, {}^{\mathcal{B}}\underline{a}_P$)



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CONSIDER A & P FIXED IN $\mathcal{B}$

$$\underline{r}_{P/O} = \underline{r}_{A/O} + \underline{r}_{P/A}$$

$$\underline{v}_P = \frac{d}{dt}\left(\underline{r}_{P/O}\right) = \frac{d}{dt}\left(\underline{r}_{A/O}\right) + \frac{d}{dt}\left(\underline{r}_{P/A}\right)$$

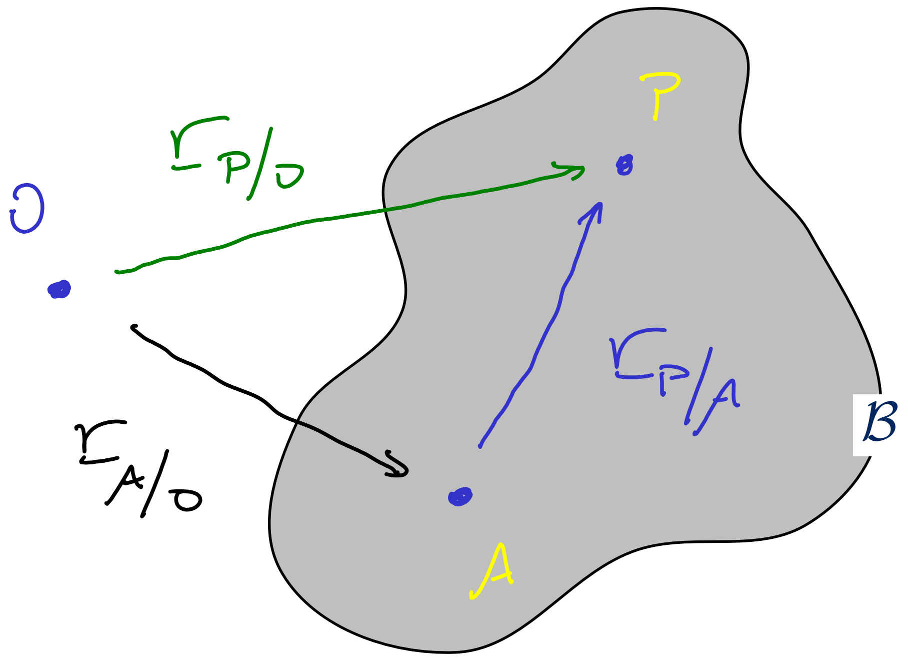
$$\boxed{\underline{v}_P = \underline{v}_A + \underline{\omega}_{\mathcal{B}} \times \underline{r}_{P/A}}$$

$$\underline{a}_P = \frac{d}{dt}\left(\underline{v}_P\right) = \frac{d}{dt}\left(\underline{v}_A\right) + \frac{d}{dt}\left(\underline{\omega}_{\mathcal{B}} \times \underline{r}_{P/A}\right)$$

$$\underline{a}_P = \underline{a}_A + \left[\frac{d}{dt}\left(\underline{\omega}_{\mathcal{B}}\right) \times \underline{r}_{P/A} + \underline{\omega}_{\mathcal{B}} \times \frac{d}{dt}\left(\underline{r}_{P/A}\right)\right]$$

$$\boxed{\underline{a}_P = \underline{a}_A + \underline{\alpha}_{\mathcal{B}} \times \underline{r}_{P/A} + \underbrace{\underline{\omega}_{\mathcal{B}} \times \left(\underline{\omega}_{\mathcal{B}} \times \underline{r}_{P/A}\right)}_{\text{CENTRIPETAL}}}$$

MUST BE EVALUATED FIRST



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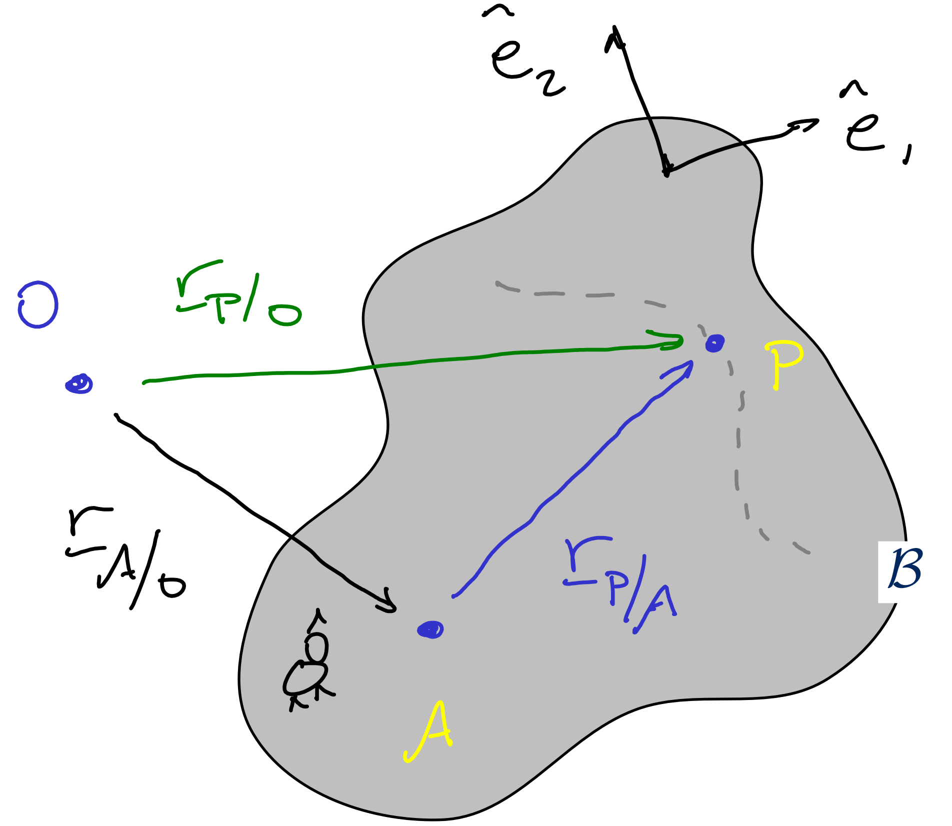
IF A IS FIXED $\mathcal{B}$, WITH P MOVING ON A PATH

$$\underline{r}_{P/O} = \underline{r}_{A/O} + \underline{r}_{P/A}$$

NOT FIXED IN $\mathcal{B}$



$$\underline{v}_P = \frac{d}{dt}(\underline{r}_{P/O}) = \frac{d}{dt}(\underline{r}_{A/O}) + \frac{d}{dt}(\underline{r}_{P/A})$$

$$\underline{v}_P = \underline{v}_A + \left[\underline{\omega}_{\mathcal{B}} \times \underline{r}_{P/A} + {}^{\mathcal{B}}\frac{d}{dt}(\underline{r}_{P/A})\right]$$

$$\boxed{\underline{v}_P = \underline{v}_A + \underline{\omega}_{\mathcal{B}} \times \underline{r}_{P/A} + {}^{\mathcal{B}}\underline{v}_P}$$

$$\underline{a}_P = \frac{d}{dt}(\underline{v}_P) = \frac{d}{dt}(\underline{v}_A) + \frac{d}{dt}(\underline{\omega}_{\mathcal{B}} \times \underline{r}_{P/A}) + \frac{d}{dt}({}^{\mathcal{B}}\underline{v}_P)$$

$$\underline{a}_P = \underline{a}_A + \underline{\alpha}_{\mathcal{B}} \times \underline{r}_{P/A} + \underline{\omega}_{\mathcal{B}} \times \left[\underline{\omega}_{\mathcal{B}} \times \underline{r}_{P/A} + {}^{\mathcal{B}}\underline{v}_P\right] + \underline{\omega}_{\mathcal{B}} \times {}^{\mathcal{B}}\underline{v}_P + {}^{\mathcal{B}}\frac{d}{dt}({}^{\mathcal{B}}\underline{v}_P)$$

$$\boxed{\underline{a}_P = \underline{a}_A + \underline{\alpha}_{\mathcal{B}} \times \underline{r}_{P/A} + \underline{\omega}_{\mathcal{B}} \times (\underline{\omega}_{\mathcal{B}} \times \underline{r}_{P/A}) + 2\underline{\omega}_{\mathcal{B}} \times {}^{\mathcal{B}}\underline{v}_P + {}^{\mathcal{B}}\underline{a}_P}$$

CENTRIPETAL CORIOLIS

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