# Planar Kinematics

## Engineering Mechanics: Dynamics

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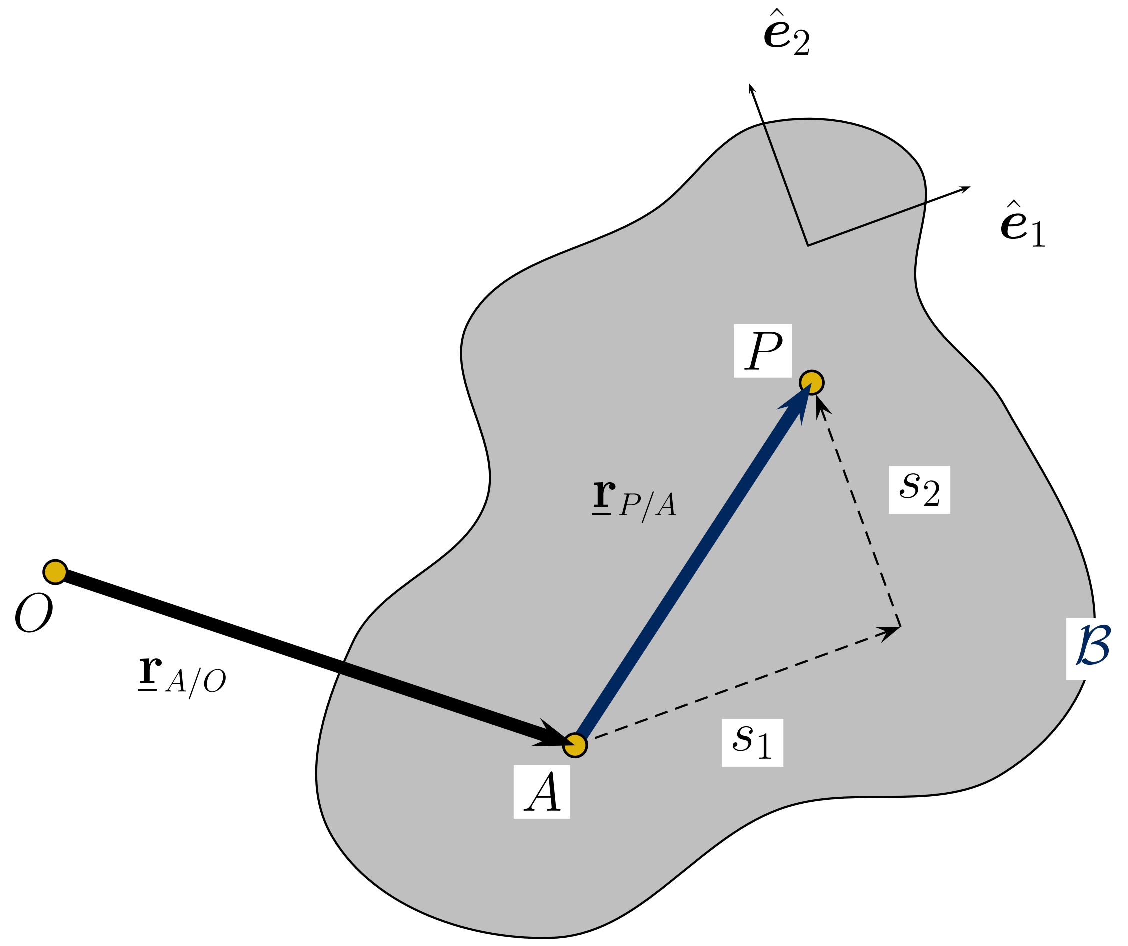
Kinematics: Consideration of the allowable motion of an object, consistent with the constraints that act on that object, without regard to the forces that produce motion or the motion that actually occurs

## Rigid bodies

- no internal deformation
- relative distance between any two points remains fixed

In terms of directions $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2)$ fixed in the object $\mathcal{B}$, the vector $\underline{\mathbf{r}}_{P/A}$ is constant

$$\underline{\mathbf{r}}_{P/A} = s_1\,\hat{\boldsymbol{e}}_1 + s_2\,\hat{\boldsymbol{e}}_2$$




The configuration of $\mathcal{B}$ can be described in terms of

- the position $\underline{\mathbf{r}}_{A/O}$ of a reference point $A$ (2 coordinates)
- the orientation of $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2)$ fixed in the object (1 coordinate)

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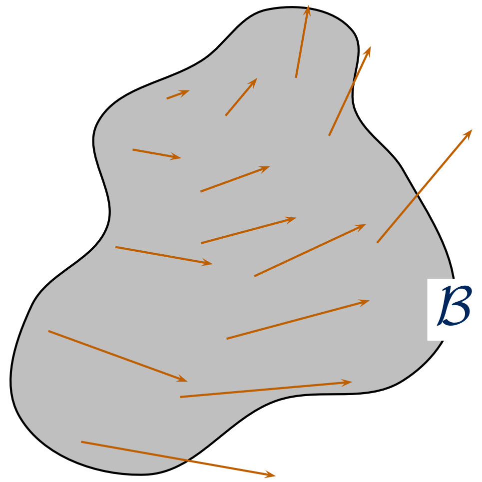



Objects can both translate and rotate, so that each point of a rigid object has a *different velocity and acceleration*.

Express the

motion of $P$ ($\underline{\mathbf{r}}_{P/O}$, $\underline{\mathbf{v}}_P$, $\underline{\mathbf{a}}_P$)

in terms of the

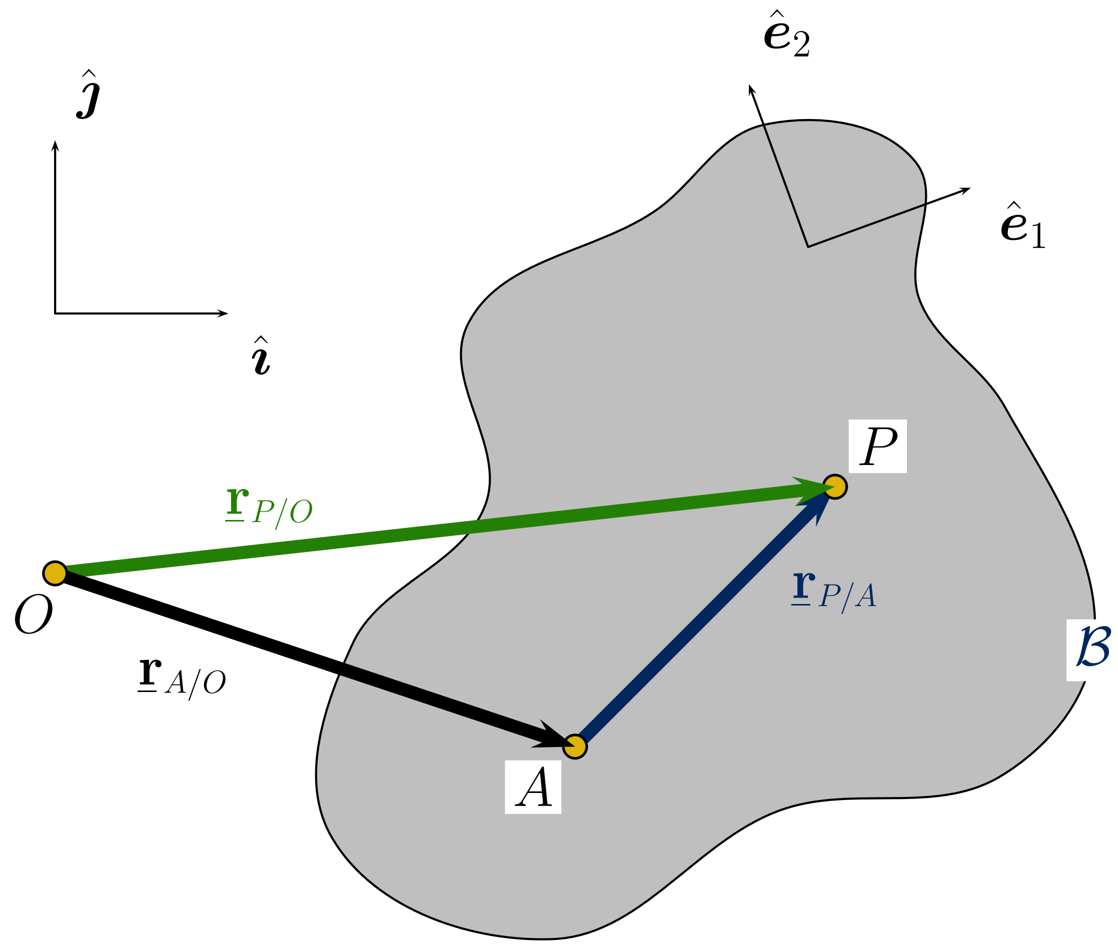
motion of $A$ ($\underline{\mathbf{r}}_{A/O}$, $\underline{\mathbf{v}}_A$, $\underline{\mathbf{a}}_A$),

and the

rotation of $\mathcal{B}$ ($\underline{\boldsymbol{\omega}}_{\mathcal{B}}$, $\underline{\boldsymbol{\alpha}}_{\mathcal{B}}$).

together with the

relative motion of $P$
with respect to $\mathcal{B}$ ($\underline{\mathbf{r}}_{P/A}$, ${}^{\mathcal{B}}\underline{\mathbf{v}}_P$, ${}^{\mathcal{B}}\underline{\mathbf{a}}_P$).

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Consider $A$ and $P$ fixed in the body $\mathcal{B}$

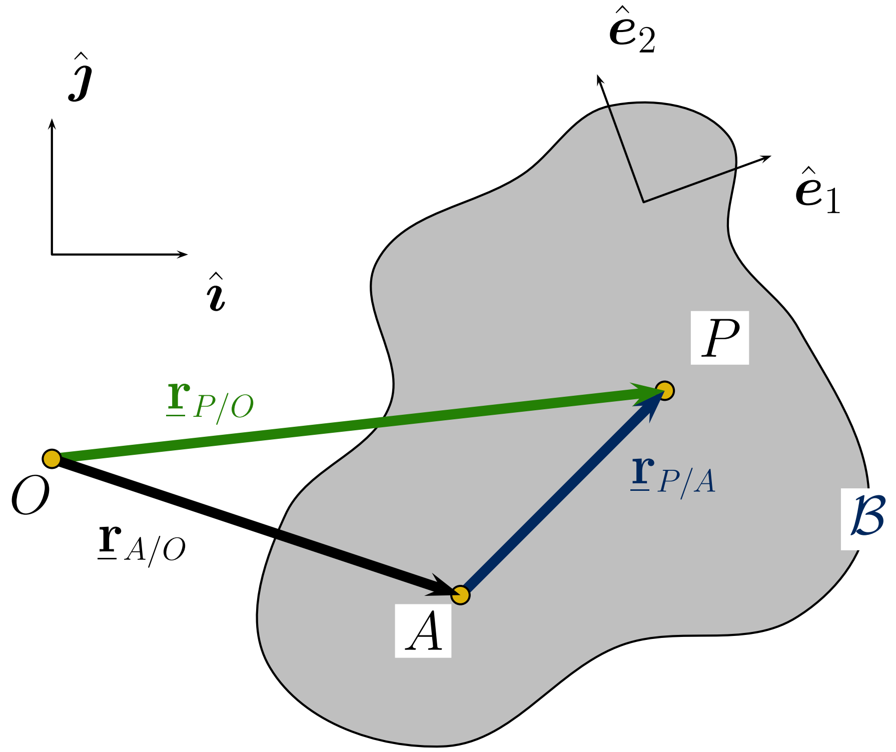
$$\underline{\mathbf{r}}_{P/O} = \underline{\mathbf{r}}_{A/O} + \underline{\mathbf{r}}_{P/A}.$$

The vector $\underline{\mathbf{r}}_{P/A}$ is fixed in the object, so that

$$\frac{d}{dt}\left(\underline{\mathbf{r}}_{P/O}\right) = \frac{d}{dt}\left(\underline{\mathbf{r}}_{A/O}\right) + \frac{d}{dt}\left(\underline{\mathbf{r}}_{P/A}\right),$$
$$\underline{\mathbf{v}}_P = \underline{\mathbf{v}}_A + \left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right),$$

likewise

$$\frac{d}{dt}\left(\underline{\mathbf{v}}_P\right) = \frac{d}{dt}\left(\underline{\mathbf{v}}_A\right) + \frac{d}{dt}\left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right),$$
$$\underline{\mathbf{a}}_P = \underline{\mathbf{a}}_A + \left[\frac{d}{dt}\left(\underline{\boldsymbol{\omega}}_{\mathcal{B}}\right) \times \underline{\mathbf{r}}_{P/A} + \underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \frac{d}{dt}\left(\underline{\mathbf{r}}_{P/A}\right)\right],$$
$$\underline{\mathbf{a}}_P = \underline{\mathbf{a}}_A + \left(\underline{\boldsymbol{\alpha}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right) + \underbrace{\left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right)\right)}_{\text{centripetal}}.$$




In the centripetal acceleration the term $\left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right)$ must be evaluated first.

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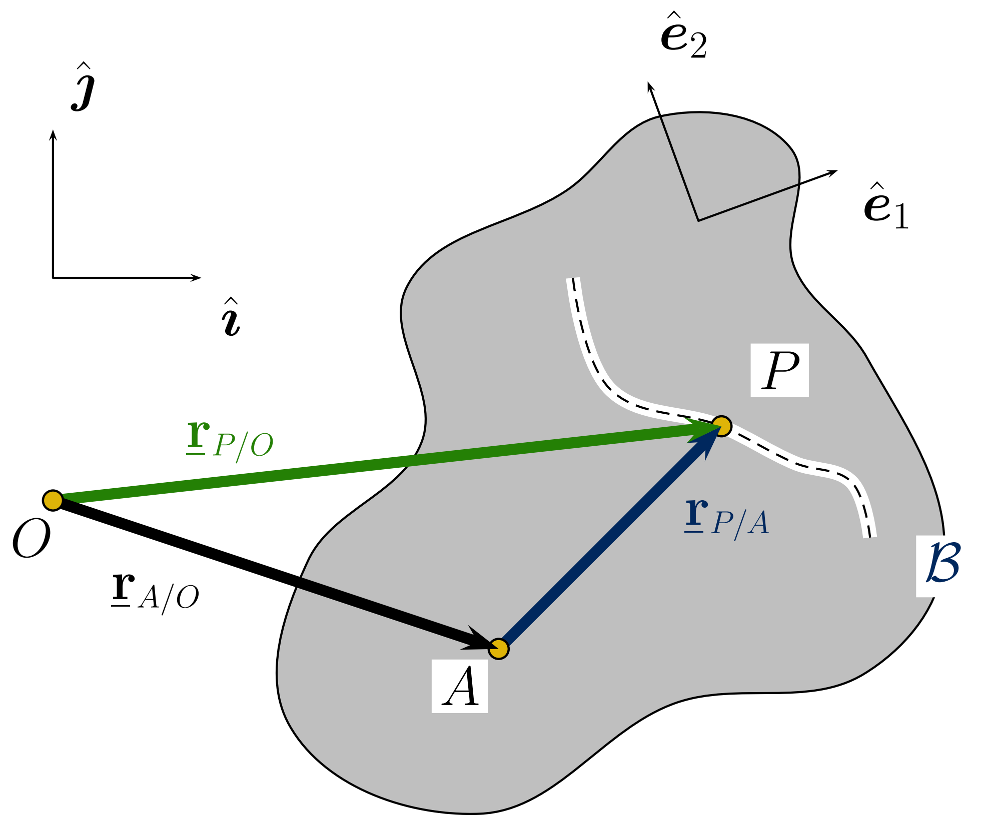
If $A$ is fixed in $\mathcal{B}$, with $P$ moving along a path in $\mathcal{B}$

$$\underline{\mathbf{r}}_{P/O} = \underline{\mathbf{r}}_{A/O} + \underline{\mathbf{r}}_{P/A}.$$

but $\underline{\mathbf{r}}_{P/A}$ is no longer fixed in $\mathcal{B}$.




$$\frac{d}{dt}\left(\underline{\mathbf{r}}_{P/O}\right) = \frac{d}{dt}\left(\underline{\mathbf{r}}_{A/O}\right) + \frac{d}{dt}\left(\underline{\mathbf{r}}_{P/A}\right),$$

$$\frac{d}{dt}\left(\underline{\mathbf{r}}_{P/O}\right) = \frac{d}{dt}\left(\underline{\mathbf{r}}_{A/O}\right) + \left[\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A} + {}^{\mathcal{B}}\frac{d}{dt}\left(\underline{\mathbf{r}}_{P/A}\right)\right],$$

$$\underline{\mathbf{v}}_P = \underline{\mathbf{v}}_A + \left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right) + {}^{\mathcal{B}}\underline{\mathbf{v}}_P.$$

For the acceleration

$$\frac{d}{dt}\left(\underline{\mathbf{v}}_P\right) = \frac{d}{dt}\left(\underline{\mathbf{v}}_A\right) + \frac{d}{dt}\left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right) + \frac{d}{dt}\left({}^{\mathcal{B}}\underline{\mathbf{v}}_P\right),$$

$$\underline{\mathbf{a}}_P = \underline{\mathbf{a}}_A + \left[\left(\underline{\boldsymbol{\alpha}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right) + \underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A} + {}^{\mathcal{B}}\underline{\mathbf{v}}_P\right)\right] + \left[\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times {}^{\mathcal{B}}\underline{\mathbf{v}}_P + {}^{\mathcal{B}}\frac{d}{dt}\left({}^{\mathcal{B}}\underline{\mathbf{v}}_P\right)\right],$$

$$\underline{\mathbf{a}}_P = \underline{\mathbf{a}}_A + \left(\underline{\boldsymbol{\alpha}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right) + \underbrace{\left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \left(\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times \underline{\mathbf{r}}_{P/A}\right)\right)}_{\text{centripetal}} + \underbrace{\left(2\,\underline{\boldsymbol{\omega}}_{\mathcal{B}} \times {}^{\mathcal{B}}\underline{\mathbf{v}}_P\right)}_{\text{Coriolis}} + {}^{\mathcal{B}}\underline{\mathbf{a}}_P.$$

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