Projectile Motion

Engineering Mechanics: Dynamics

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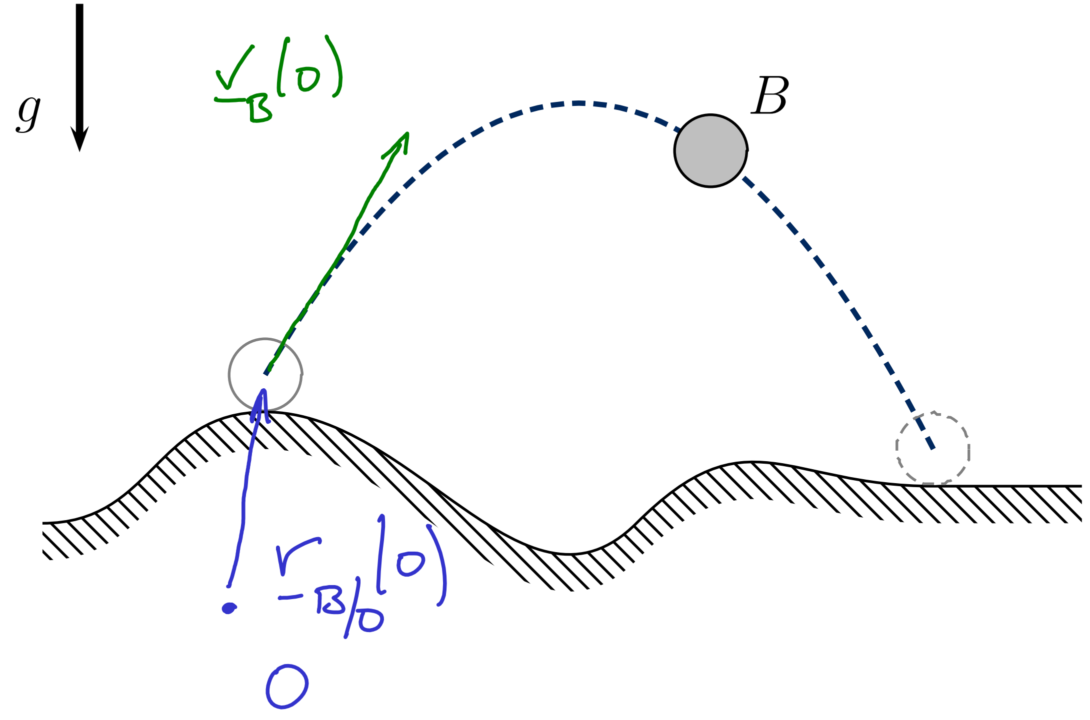
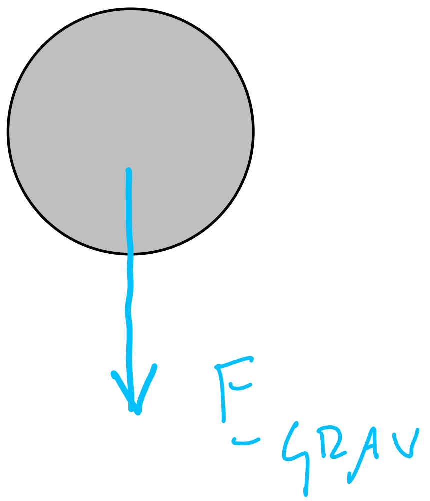
- Describes the motion of a particle subject only to the force due to gravity.

TRAJECTORY ONLY DEPENDS ON THE INITIAL CONDITIONS...



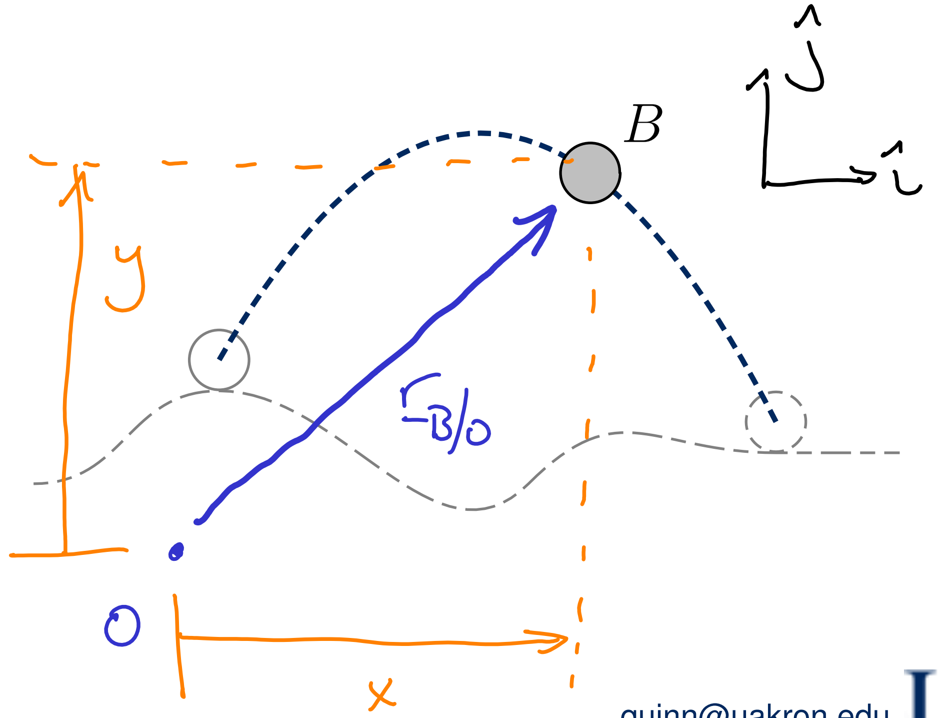
Coordinates and Directions



ONLY FORCE ARISES FROM GRAVITY

ASSUME PLANAR MOTION

$$\underline{r}_{B/O}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$



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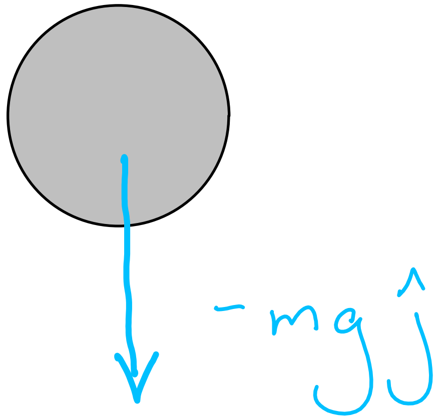
Kinematics/Free Body Diagram

ACCELERATION

$$\underset{\sim}{a}_B(t) = \ddot{x}(t)\,\hat{\imath} + \ddot{y}(t)\,\hat{\jmath}$$



Equations of Motion

LINEAR MOMENTUM BALANCE

$$-mg\hat{\jmath} = \sum \underset{\sim}{F} = m\underset{\sim}{a}_B = m(\ddot{x}\,\hat{\imath} + \ddot{y}\,\hat{\jmath})$$

$$\underset{\sim}{0} = [m\ddot{x}]\,\hat{\imath} + [m\ddot{y} + mg]\,\hat{\jmath}$$

TAKING COMPONENTS

$\hat{\imath}$: $\cancel{m}\ddot{x} = 0$

$\hat{\jmath}$: $\cancel{m}\ddot{y} + \cancel{m}g = 0$

$$\boxed{\begin{aligned}\ddot{x} &= 0\\ \ddot{y} &= -g\end{aligned}}$$

(CONSTANT ACCELERATION)

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INTEGRATE TO FIND VELOCITY

$$\ddot{x} = \frac{d^2}{dt^2}(x) = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$\int_0^t \left\{ \ddot{x}(\tau) = 0 \right\} d\tau \longrightarrow \dot{x}(t) - \dot{x}(0) = 0$$

$$\dot{x}(t) = v_x \qquad (\dot{x}(0) \equiv v_x)$$

$$\int_0^t \left\{ \ddot{y}(\tau) = -g \right\} d\tau \longrightarrow \dot{y}(t) - \dot{y}(0) = -gt$$

$$\dot{y}(t) = -gt + v_y \qquad (\dot{y}(0) \equiv v_y)$$

INTEGRATE TO FIND POSITION

$$\int_0^t \left\{ \dot{x}(\tau) = v_x \right\} d\tau \longrightarrow x(t) - x(0) = v_x t$$

$$x(t) = v_x t + x_0 \qquad (x(0) \equiv x_0)$$

$$\int_0^t \left\{ \dot{y}(\tau) = -g\tau + v_y \right\} d\tau \longrightarrow y(t) - y(0) = -\frac{g}{2}t^2 + v_y t$$

$$y(t) = -\frac{g}{2}t^2 + v_y t + y_0 \qquad (y(0) \equiv y_0)$$

VALID FOR ANY PARTICLE UNDERGOING PROJECTILE MOTION

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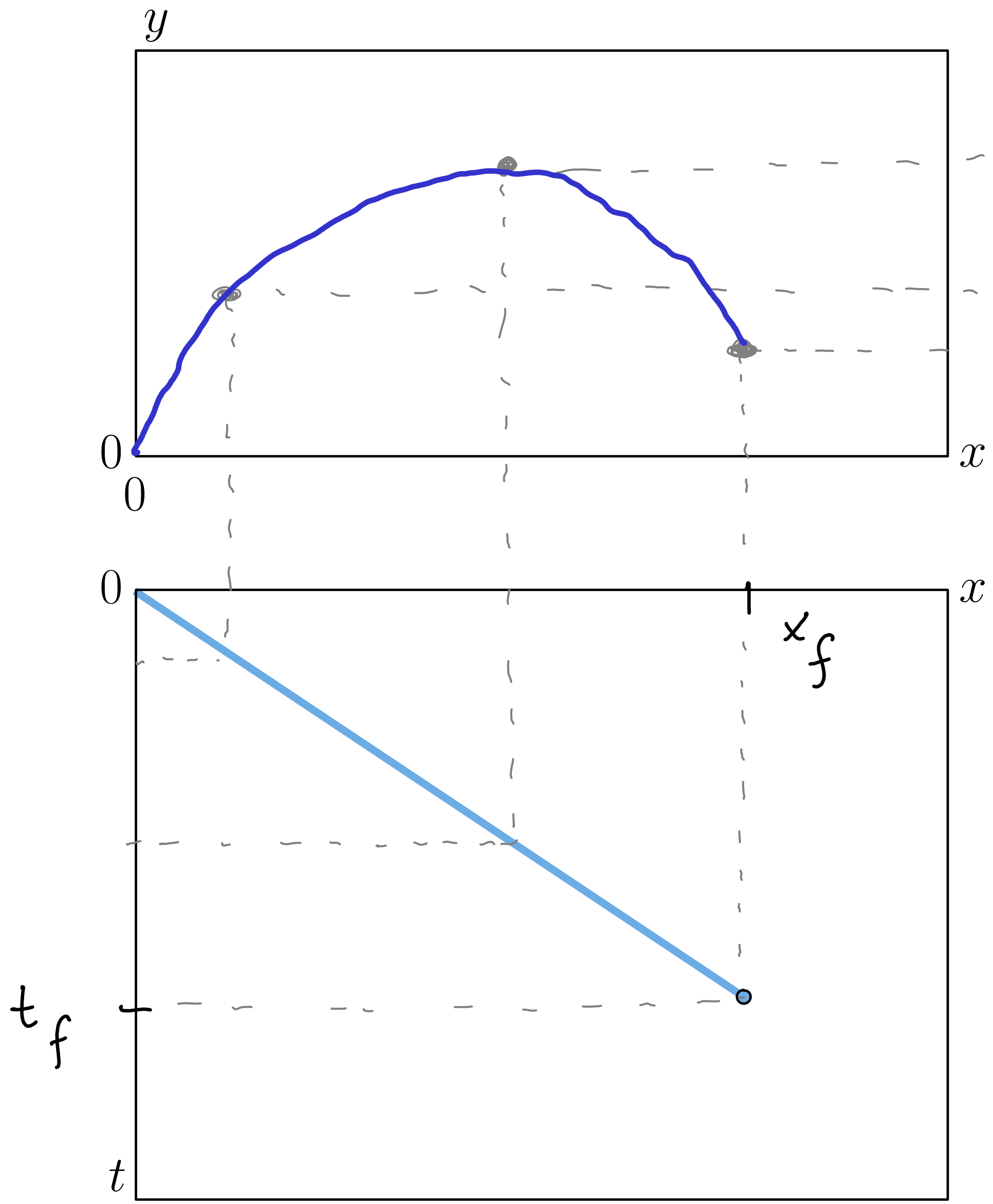

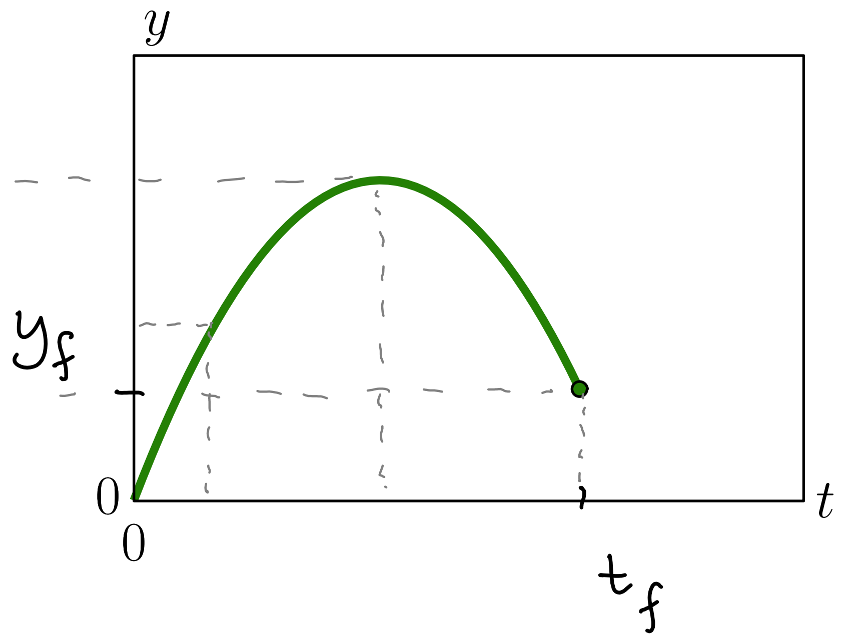

$$(x_0, y_0) = (0, 0)$$

$$x(t) = v_x t$$

$$\downarrow$$

$$t = \frac{x}{v_x}$$

$$y(t) = -\frac{g}{2}t^2 + v_y t$$

$$\downarrow$$

$$y(x) = -\frac{g}{2}\left(\frac{x}{v_x}\right)^2 + v_y\left(\frac{x}{v_x}\right)$$

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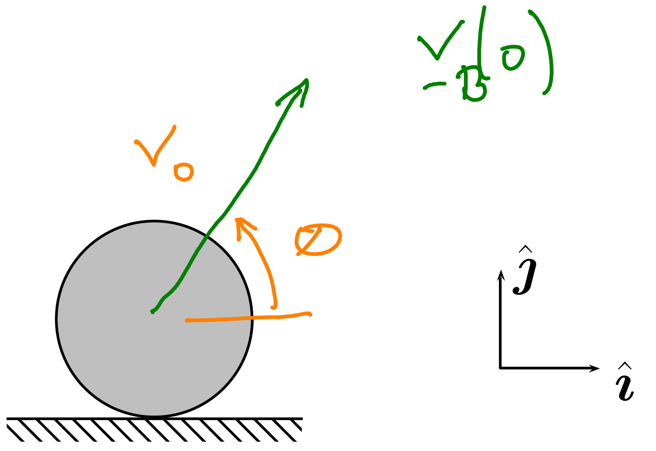
INITIAL VELOCITY

$$\underline{v}_B(0) = v_x\hat{i} + v_y\hat{j} = v_0\left(C_\phi\hat{i} + S_\phi\hat{j}\right)$$



$$\dot{x}(0) \equiv v_x = v_0 C_\phi \qquad \dot{y}(0) \equiv v_y = v_0 S_\phi$$

IN TERMS OF v_0 & ϕ

$$\dot{x}(t) = v_0 C_\phi \qquad \dot{y}(t) = -gt + v_0 S_\phi \qquad (x_0, y_0) = (0,0)$$

$$x(t) = v_0 C_\phi \cdot t \qquad y(t) = -\frac{g}{2}t^2 + v_0 S_\phi \cdot t$$

$$y(x) = \frac{-gx^2}{2v_x^2} + \frac{v_y x}{v_x} = \frac{-gx^2}{2v_0^2 C_\phi^2} + T_\phi x$$

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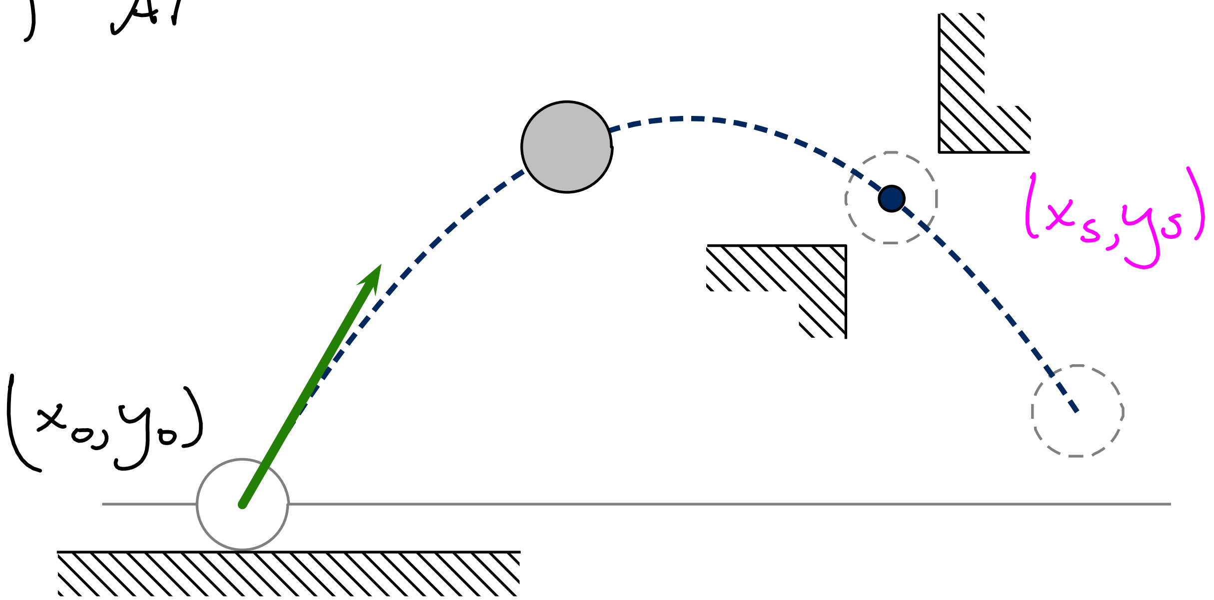
The motion can also be defined in terms of auxiliary conditions, rather than the initial conditions.

AUXILIARY CONDITIONS PROVIDE EQUATIONS THAT CAN DETERMINE THE INITIAL CONDITIONS

Secondary Position

TRAJECTORY PASSES THROUGH (x_s, y_s) AT TIME t_s



$$x(t_s) = x_s = v_x \cdot t_s + x_0$$

$$y(t_s) = y_s = -\frac{g}{2} t_s^2 + v_y t_s + y_0$$

$$\longrightarrow \quad y_s = -\frac{g}{2}\left(\frac{x_s - x_0}{v_x}\right)^2 + v_y \frac{(x_s - x_0)}{v_x} + y_0$$

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Maximum Height

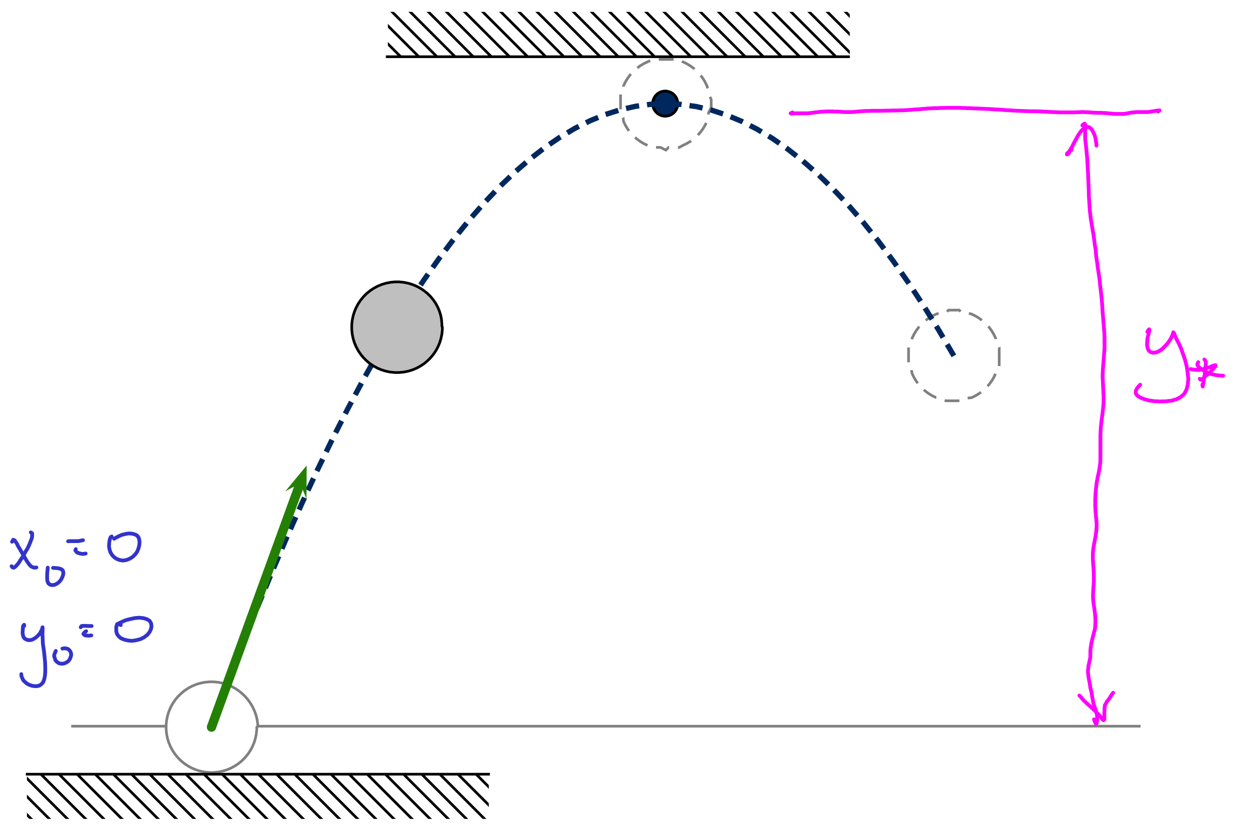
MAXIMUM HEIGHT $y = y_*$ OCCURS AT TIME $t = t_*$

$$\dot{y}(t_*) = 0 = -g t_* + v_y \longrightarrow t_* = \frac{v_y}{g}$$

SO THAT

$$y(t_*) = y_* = -\frac{g}{2}\left(\frac{v_y}{g}\right)^2 + v_y\left(\frac{v_y}{g}\right)$$

$$= \frac{v_y^2}{2g} = \frac{v_0^2 C_\theta^2}{2g}$$



$$x(t_*) = v_x t_* = \frac{v_x v_y}{g} = \frac{v_0^2 S_\theta C_\theta}{2g}$$

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