Vectors

Engineering Mechanics: Dynamics

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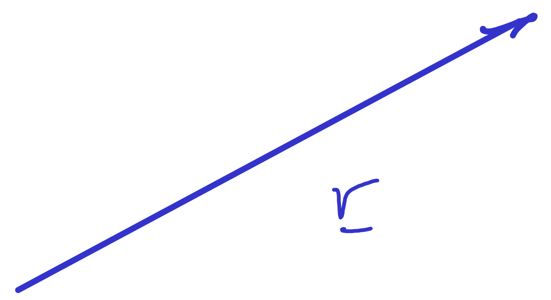
The elements of dynamics (e.g. position, acceleration, forces, moments) are described by *vectors*.

Vector: a mathematical quantity that describes multiple independent variables

- used to describe physical objects with magnitude and direction
- obey laws of vector addition and scalar multiplication
- follow additional operations including dot and cross products

VECTOR CAN BE DEFINED

- GEOMETRICALLY
- INDEPENDENT OF HOW WE CHOOSE TO MEASURE IT



VECTORS ARE DENOTED BY UNDERBARS $\underline{r}$

EXCEPT FOR UNIT VECTORS (LENGTH ONE)

DENOTED WITH HAT $\hat{\imath}$

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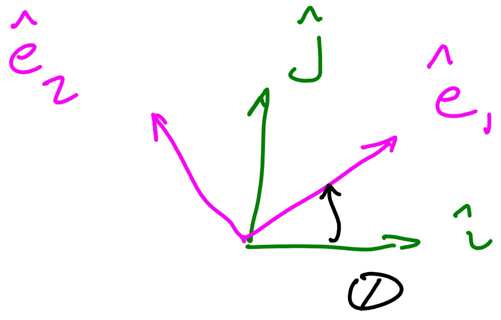
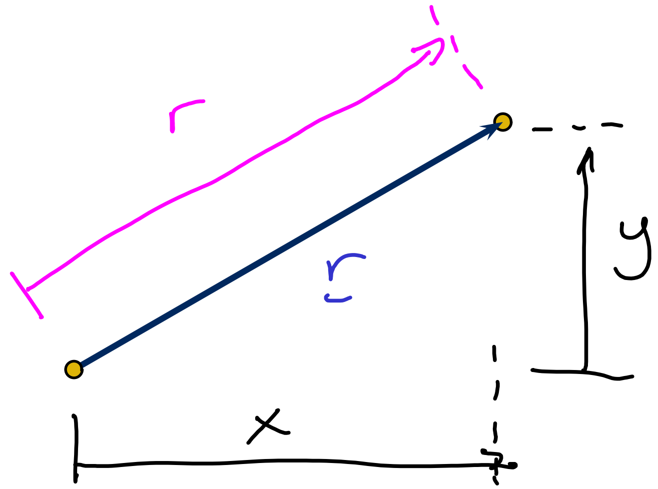
Vectors are specified with respect to known *basis directions*.

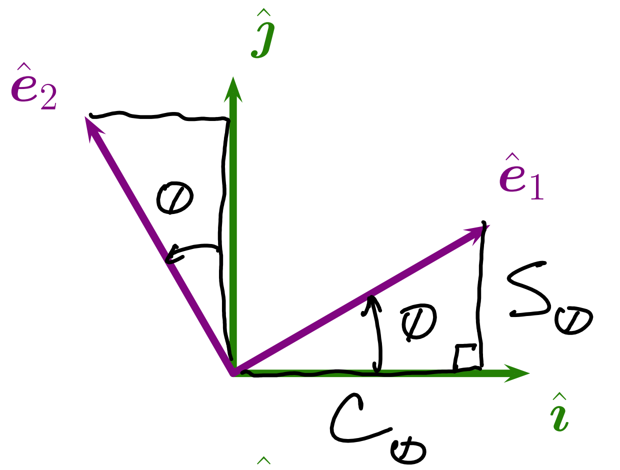
$(\hat{\imath}, \hat{\jmath})$: CARTESIAN DIRECTIONS

$(\hat{e}_1, \hat{e}_2)$: POLAR DIRECTIONS





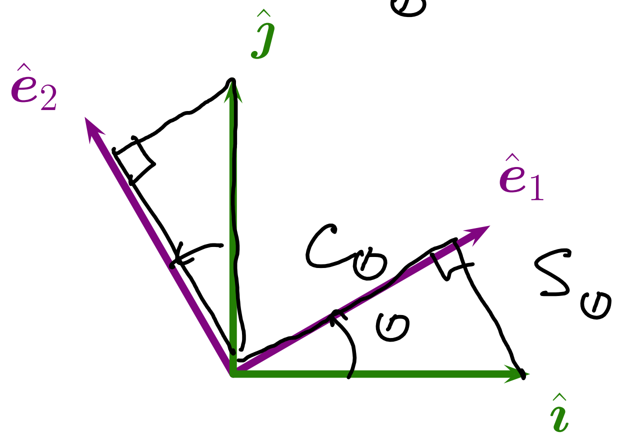
$$\underline{r} = x\hat{\imath} + y\hat{\jmath} = r\hat{e}_1$$



$$\cos\theta \equiv C_\theta$$
$$\sin\theta \equiv S_\theta$$

$$\hat{e}_1 = C_\theta \hat{\imath} + S_\theta \hat{\jmath}$$
$$\hat{e}_2 = -S_\theta \hat{\imath} + C_\theta \hat{\jmath}$$

$$\hat{\imath} = C_\theta \hat{e}_1 - S_\theta \hat{e}_2$$
$$\hat{\jmath} = S_\theta \hat{e}_1 + C_\theta \hat{e}_2$$



$$\begin{aligned}\underline{r} = x\hat{\imath} + y\hat{\jmath} &= r\hat{e}_1 \\ &= r\left(C_\theta \hat{\imath} + S_\theta \hat{\jmath}\right) \\ &= (rC_\theta)\hat{\imath} + (rS_\theta)\hat{\jmath}\end{aligned}$$

$$x = rC_\theta$$
$$y = rS_\theta$$

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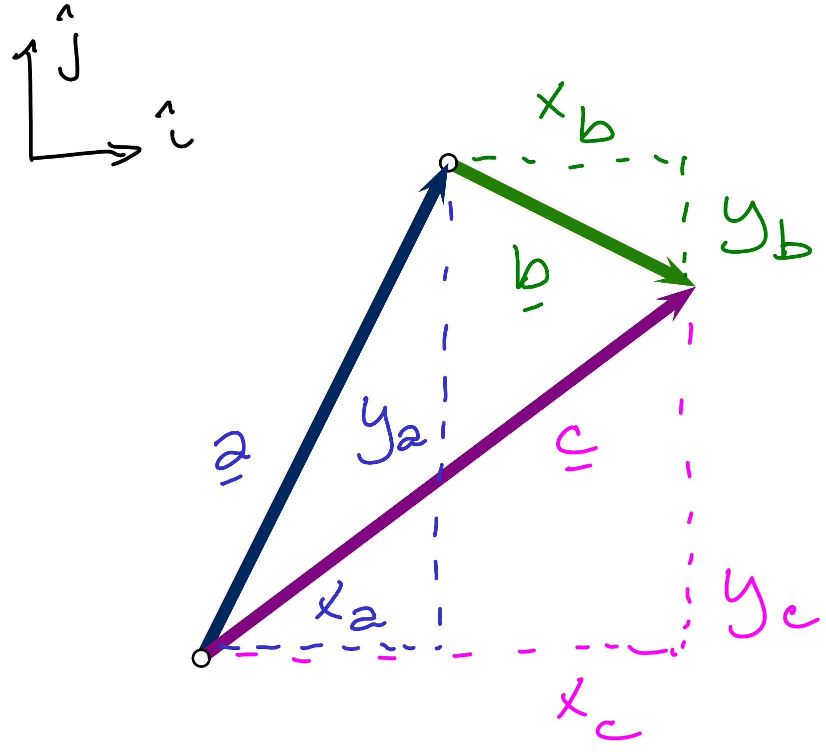


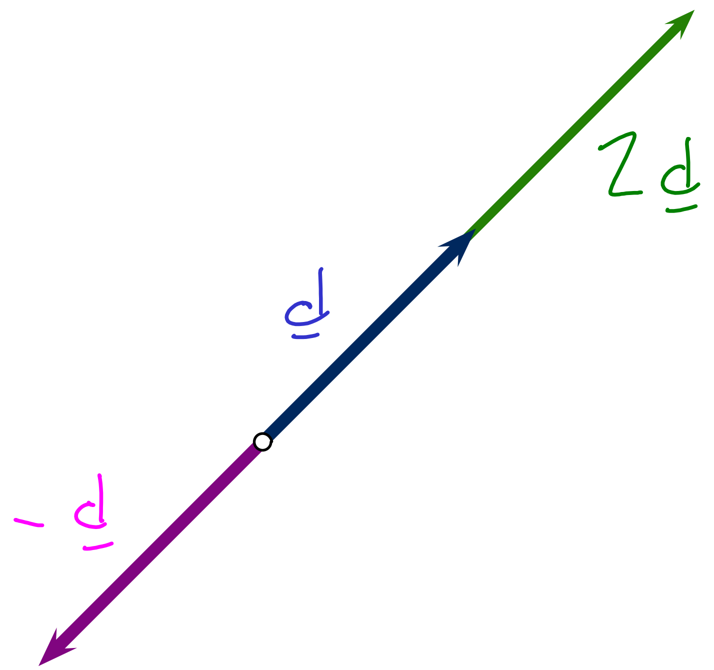
Vector Addition

$$\underline{c} = \underline{a} + \underline{b}$$

$$\underline{c} = x_c \hat{i} + y_c \hat{j} \qquad \underline{a} = x_a \hat{i} + y_a \hat{j} \qquad \underline{b} = x_b \hat{i} + y_b \hat{j}$$

$$\left(x_c \hat{i} + y_c \hat{j} \right) = \left(x_a \hat{i} + y_a \hat{j} \right) + \left(x_b \hat{i} + y_b \hat{j} \right)$$

$$\longrightarrow \quad x_c = x_a + x_b, \qquad y_c = y_a + y_b$$



Scalar multiplication

$$\underline{d} = x_d \hat{i} + y_d \hat{j} \longrightarrow \quad 2\underline{d} = 2x_d \hat{i} + 2y_d \hat{j}$$

$$-\underline{d} = -x_d \hat{i} - y_d \hat{j}$$

LEAVES THE DIRECTION UNCHANGED

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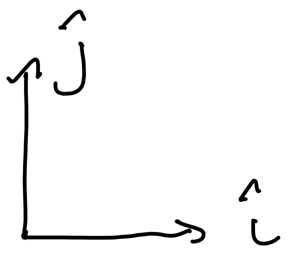
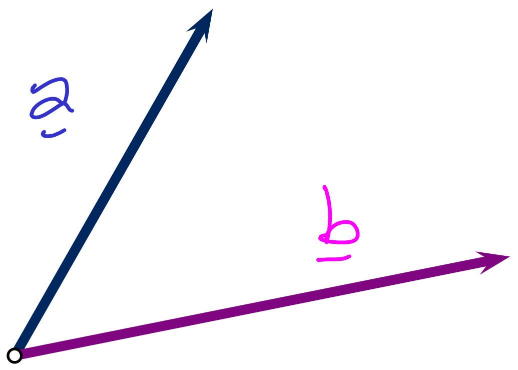
Dot Product

OPERATES ON TWO VECTORS, GIVES A SCALAR

PERPENDICULAR VECTORS $\longrightarrow$ 0

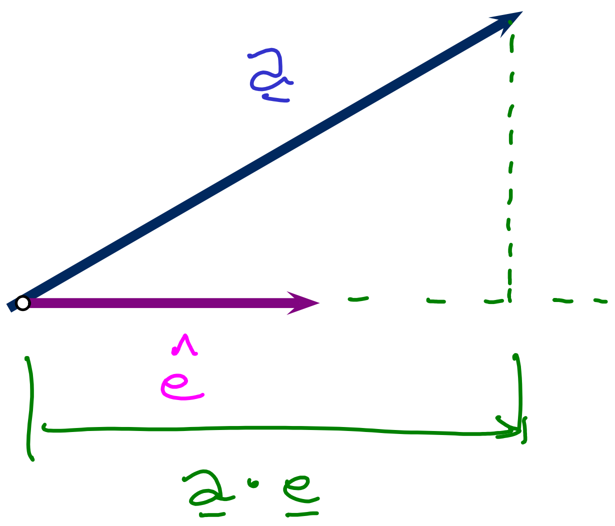


$$\hat{i}\cdot\hat{i} = 1 \qquad \hat{i}\cdot\hat{j} = 0$$
$$\hat{j}\cdot\hat{j} = 1 \qquad \hat{j}\cdot\hat{i} = 0$$



$$\underline{a} = x_a\hat{i} + y_a\hat{j}$$
$$\underline{b} = x_b\hat{i} + y_b\hat{j}$$

$$\underline{a}\cdot\underline{b} = (x_a\hat{i} + y_a\hat{j})\cdot(x_b\hat{i} + y_b\hat{j})$$
$$= x_a x_b \overbrace{(\cancel{\hat{i}\cdot\hat{i}})}^{1} + y_a x_b \overbrace{(\cancel{\hat{j}\cdot\hat{i}})}^{0}$$
$$+ x_a y_b \overbrace{(\cancel{\hat{i}\cdot\hat{j}})}^{0} + y_a y_b \overbrace{(\cancel{\hat{j}\cdot\hat{j}})}^{1} = x_a x_b + y_a y_b$$
$$= \underline{b}\cdot\underline{a}$$



$\underline{a}\cdot\hat{e} \longrightarrow$ PROJECTION OF $\underline{a}$ IN THE $\hat{e}$ DIRECTION

MAGNITUDE

$$\|\underline{a}\| = \sqrt{\underline{a}\cdot\underline{a}} = \sqrt{x_a^2 + y_a^2}$$

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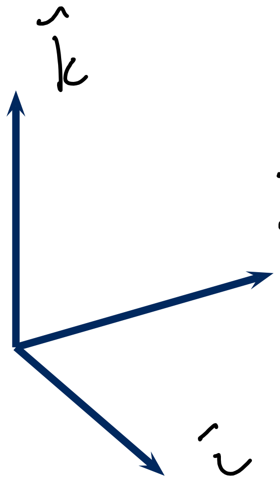
Cross Product

VECTOR CROSS PRODUCT OPERATES ON VECTORS, PRODUCES A THIRD VECTOR



$$\hat{i}\times\hat{i} = \underline{0} \qquad \hat{j}\times\hat{i} = -\hat{k} \qquad \hat{k}\times\hat{i} = \hat{j}$$

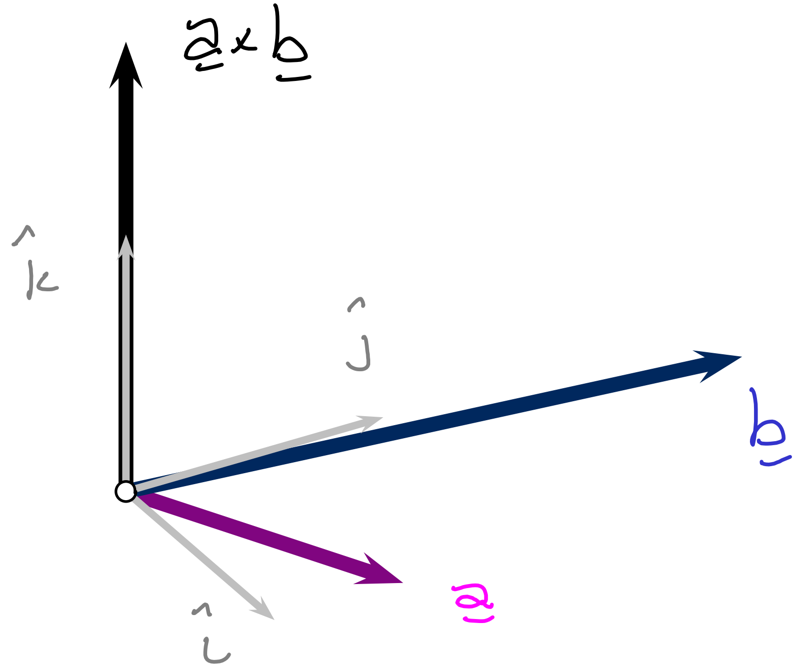
$$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{j} = \underline{0} \qquad \hat{k}\times\hat{j} = -\hat{i}$$

$$\hat{i}\times\hat{k} = -\hat{j} \qquad \hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{k} = \underline{0}$$

$\longrightarrow$ +

$\hat{i} \quad \hat{j} \quad \hat{k} \quad \hat{i} \quad \hat{j}$

− $\longleftarrow$



$$\underline{a} = x_a\hat{i} + y_a\hat{j}, \quad \underline{b} = x_b\hat{i} + y_b\hat{j}$$

$$\underline{a}\times\underline{b} = \left(x_a\hat{i} + y_a\hat{j}\right)\times\left(x_b\hat{i} + y_b\hat{j}\right)$$

$$= x_a x_b\cancel{(\hat{i}\times\hat{i})}^{\underline{0}} + x_a y_b\underbrace{\cancel{(\hat{i}\times\hat{j})}}_{\hat{k}} + y_a x_b\underbrace{\cancel{(\hat{j}\times\hat{i})}}_{-\hat{k}} + y_a y_b\cancel{(\hat{j}\times\hat{j})}^{\underline{0}}$$

$$= \left(x_a y_b - y_a x_b\right)\hat{k} = -\underline{b}\times\underline{a}$$

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